Names of Some Really Big Numbers

> Words of wisdom are spoken by children at least as often as by scientists. The name “googol” was invented by a child (Dr. Kasner’s nine-year-old nephew, Milton Sirotta) who was asked to think up a name for a very big number, namely, 1 with a hundred zeros after it. He was very certain that this number was not infinite, and therefore equally sure that it had to have a name. At the same time that he suggested “googol” he gave a name for a still larger number: “Googolplex.”[1]
>
> **Edward Kasner** (American Mathematician; 1878 - 1955)
> **James Newman** (American Mathematician and Lawyer; 1907 - 1966)

A Trillion Dollars in Human Terms

The news regularly reports on the US National Debt, which is in the trillions of dollars. For most people, this number is just too big to conceive. In this investigation, you will place a human context to this amount of money.

The population of several states is given in the table on the next page. You probably have friends and/or relatives who live in these states.

[1] From Mathematics and the Imagination, Simon and Schuster, 1940.

State	Population (2010)
Connecticut	3,574,097
Maine	1,328,361
New Hampshire	1,316,470
Rhode Island	1,052,567
Vermont	625,741

For each of the following questions find the desired quantities and give a single value for the entirety of the five states considered together.

1. If we consider the average family size to be 4, how many total families are there in these states?

2. What would the total cost be to build a $100,000 house for each of the families in these states?

3. What would the total cost be to give each of the families in these states a $15,000 car?

4. What would the total cost be to build a $5 million library and $15 million hospital in a total of 250 cities in these states?

5. What would the total cost be to build a $10 million school in a total of 500 cities in these states?

6. If you started with $1 trillion dollars and did all of the things described in Investigations 2326, how much money, if any, would be left?

7. If you put the leftover balance calculated in Investigation 27 in the bank and earned 3% interest per year, compounded yearly, how much interest would be earned each year? [Assume the interest is not reinvested.]

8. If you used the yearly interest from Investigation 28 to pay doctors, nurses, teachers, police officers, and firefighters each $40,000 a year salaries, how many such public servants could you hire with the interest alone?

9. Using Investigation 29, how many doctors, nurses, teachers, police officers, and firefighters would the interest provide for each of 250 different cities in these states?

All of the costs you considered in Investigations 1-9 cost less than $1 trillion dollars. In comparison, in each of the years 2002 - 2010 the United State national debt grew at least one-half a trillion dollars.

- At 21 Jan 2016 at 02:41:42 PM GMT, the debt was $18,944,681,555,689.43.
- Exactly one minute later, the debt had grown by another $2,655,149.47![2]

10. Find some information on the U.S. national debt currently and relate what you find to the discussion of large numbers in a brief essay of a few paragraphs.

Names of Some Really Large Numbers

One trillion is pretty large in human terms, as you have just seen. But it is dwarfed by a **googol** - a number named by a kid. Do other really large numbers have names, like a “zillion” perhaps? The majority of large number names that are commonly used, words like million, billion, and trillion, were named systematically by Nicolas Chuquet (French Mathematician; 1445 - 1488) circa 1484.

Name	Scientific Notation	
Million	10^{6}	1,000,000
Billion	10^{9}	1,000,000,000
Trillion	10^{12}	1,000,000,000,000
Quadrillion	10^{15}	1,000,000,000,000,000
Quintillion	10^{18}	1,000,000,000,000,000,000
Sextillion	10^{21}	1,000,000,000,000,000,000,000
Septillion	10^{24}	1,000,000,000,000,000,000,000,000
Octillion	10^{27}	1,000,000,000,000,000,000,000,000,000
Nonillion	10^{30}	1,000,000,000,000,000,000,000,000,000,000
Decillion	10^{33}	1,000,000,000,000,000,000,000,000,000,000,000

Over time these names were extended, but the most remarkable extension came just recently when a new scheme was adapted to include numbers like *millinilliontrillion* by Allan Wechsler, John H. Conway in the 1990’s.[3]

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[2] There are a number of debt clocks online, including http://www.brillig.com/debt_clock/.

The table below gives you a glimpse of some of the fabulous names. Some specific numbers which had previously been named, like googol, googolplex and Skewes', can be named by their original name or their name in this scheme depending on context. Googol's name in this scheme, ten Duotrigintillion, does not roll off the tongue quite as nicely as googol.

Thousand	1,000	10^3
Million	1,000,000	10^6
Billion	1,000,000,000	10^9
Trillion	1,000,000,000,000	10^{12}
Quadrillion	1,000,000,000,000,000	10^{15}
Quintillion	1,000,000,000,000,000,000	10^{18}
Tredecillion	$1,\underbrace{000,...,000}_{42 \text{ zeroes}}$	10^{42}
Googol	$1,\underbrace{000,000,...,000,000}_{100 \text{ zeroes}}$	10^{100}
Octdecitrecentillion	$1,\underbrace{000,000,000,...,000,000,000}_{957 \text{ zeroes}}$	10^{957}
Millinillitrillion	$1,\underbrace{000,000,000,000,...,000,000,000,000}_{3,000,012 \text{ zeroes}}$	$10^{3,000,012}$
Googolplex	$1,\underbrace{000,000,000,000,000,...,000,000,000,000,000}_{10^{100} \text{ zeroes}}$	$10^{10^{100}}$
Skewes' Number	$1,\underbrace{000,000,000,000,000,000,...,000,000,000,000,000,000}_{10^{10^{34}} \text{ zeroes}}$	$10^{10^{10^{34}}}$

Exponential Growth and Really Large Numbers

We'd like to construct some situations where really large numbers arise.

Investigation

11. Take a blank sheet of 8 1/2" by 11" paper and fold it in half. Now fold it in half again. And again. See how many such folds you can make, describing what limitations you face.

3 See http://mrob.com/pub/math/largenum.html#conway-wechsler for a more complete explanation.

Britney Gallivan and the Paper-Folding Myth

Urban legend has it that the maximum number of times you can fold a piece of paper in half is seven. Like many things, myths become “fact” when they hit the Internet - see the PBS entry [http://pbskids.org/zoom/activities/phenom/paperfold.html.](http://pbskids.org/zoom/activities/phenom/paperfold.html) Like many such “truths”, they are false.

In December, 2001 Britney Gallivan (American Student; 1985 -) challenged this legend by analyzing paper-folding mathematically and finding limits on the number of folds based on the length, L, and thickness, t, of the material used. What she found is that in the limiting case the number of folds, n, requires a material whose minimum length is

$$L = \frac{\pi \cdot t}{6}(2^n + 4)(2^n - 1)$$

Using this result she was able to fold paper with 11 folds and later with 12 folds.[4] Subsequently, Gallivan received quite a bit of noteriety: her story was mentioned in an episode of the CBS show *Numb3rs*, was included on an episode of The Discovery Channel’s *Myth Busters*, and resulted in her being invited to present the keynote address at the regional meeting of the National Council of Teachers of Mathematics in Chicago in September, 2006.

Investigation, continued

12. After your first fold, how many layers thick is your folded paper?

13. After your second fold, how many layers thick is your folded paper?

14. After your third fold?

15. Continue a few more times until you see a clear pattern forming. Describe this pattern.

16. Suppose you folded your paper 20 times; how many layers thick would it be?

17. Suppose you folded your paper 30 times; how many layers thick would it be?

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[4] See http://pomonahistorical.org/12times.htm for more information.

18. Use Investigations 12-16 to determine a closed-term algebriac expression for the number of layers thick your folded paper will be after n folds.

19. How tall, in an appropriate unit of measure, is a pile of paper that has as many layers as the folded paper you have described in Investigation 17? (Hint: a ream of paper, which is 500 sheets, is about 2 1/2” tall.)

20. How tall, in an appropriate unit of measure, is a pile of paper that has as many layers as the folded paper you have described in Investigation 42?

21. How many folds are necessary for the number of layers to exceed a googol? Explain.

This paper folding example illustrates what is known as exponential growth because a quantity grows by a fixed rate at each stage; i.e. we repeatedly multiply at each stage. We see it yields gigantic numbers. If we want, we can built fantastically larger numbers by repeatedly exponentiating at each stage.

If we were to add the numbers below to the table of big numbers on page 4, where would each of the following numbers go?[1]

22. 10^{10}

23. $10^{10^{10}}$

24. $10^{10^{10^{10}}}$

25. Is there any limit to the process described in Investigations 22-24? Explain.

26. Archimedes worked very hard to determine a way to write very large numbers. What do you think his opinion of our modern notational systems might be?

27. On 23 August, 2008 a group using a distributed computing program available through the Great Internet Mersenne Prime Search (GIMPS) found the largest known prime number, $2^{43,112,609}-1$, which is a whopping 12,978,189 digits long. As the discoverer, GIMPS was awarded $100,000 by the Electronic Frontier Foundation for finding the first prime number of more than 10,000,000 digits.

Where does the Mersenne prime just described belong in the table of large numbers above?