Mathemagical Tricks

> Proof is an idol before which the mathematician tortures himself.
>
> **Sir Arthur Eddington**
> (British astrophysicist; 1882 - 1944)

1. Choose two single-digit numbers. Then perform the following computations in order:
   - Multiply the first number by 2
   - Add 3 to the result
   - Multiply the sum by 5
   - Add the second number to the product
   - Multiply the sum by 10.

Show these computations step by step and write down the end result.

A mathemagician - human in the form of your teacher or a peer, will now divine the identity of your two numbers from the result of your computation.

2. In an effort to understand how this trick worked, compile a list of beginning numbers and the final computations.

3. From this list in Investigation 2, can you determine how it was that the mathemagician divined the two numbers in question? Explain.

4. It wouldn’t be much of a trick if it only worked sometimes. Use algebra to prove that this trick will work for any pair of beginning numbers.
Here’s another trick.

5. Choose a secret number. Then perform the following computations:

- Add 1 to the number chosen
- Multiply the sum by 3
- Add the square of the original number to this product
- Multiply the sum by 4
- Subtract 3 from the product
- Take the square root of the difference.

Show these computations step by step and write down the end result.

A mathemagician - human in the form of your teacher or a peer will now divine the identity of your two numbers from the result of your computation.

6. Do you think this is a compelling trick? Explain.

7. In an effort to understand how this trick worked, compile a list of beginning numbers and the final computations.

8. From this list in Investigation 7, can you determine how it was that the mathemagician divined the two numbers in question? Explain.

9. It wouldn’t be much of a trick if it only worked sometimes. Use algebra to prove that this trick will work for any beginning number.

Algebraic Proofs

When you solve equations you use the Properties of Equality.

Summary **Properties of Equality**

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then b can replace a in any expression.

10. Support each conclusion with a reason.

a. Given: $6x + 2 = 12$
Conclusion: $6x = 10$ Reason:___________________________

b. Given: $q - x = r$
Conclusion: $4(q - x) = 4r$ Reason:___________________________

c. Given: $5(y - x) = 20$ Reason:___________________________

Conclusion: $5y - 5x = 20$

To create a Two column proof, simply arrange a series of statements and reasons. The term **Given** is used to identify the original algebraic statement, or other given information.

11. Fill in the reasons that justify each statement to complete the following two column proof:

Statement	**Reason**

1. $9x + 3 = 17$	1. Given
2. $9x = 36$	2.
3. $x = 4$	3.

12. Fill in the reasons that justify each statement to complete the following two column proof:

Statement	Reason
1. $2(x - 12)$ =40	1. Given
2. $2x - 24 = 40$	2.
3. $2x = 64$	3.
4. $x = 32$	4.

13. Solve the following equations and justify your solution in a two-column proof:

a. $7x + 2 = 23$

Statement	Reason

b. $\frac{2}{3}y + 6$ =14

Statement	Reason

c. $5a - 2 = 3a + 17$

Statement	**Reason**

d. $2(x - 5) = 3(x + 7)$

Statement	**Reason**