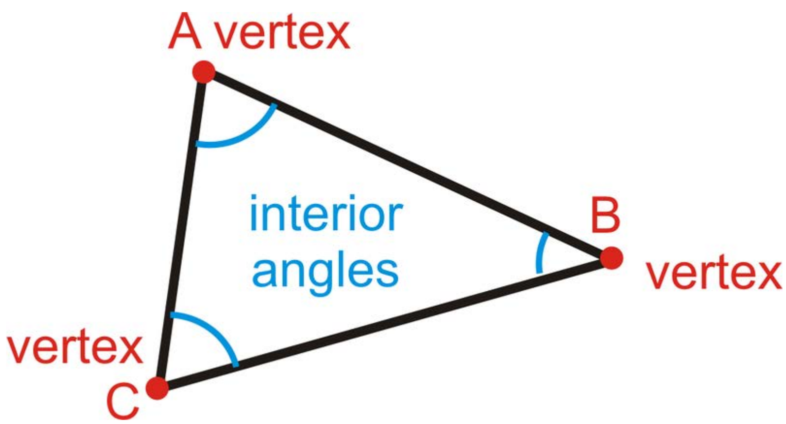
Triangle Proofs

One of the fundamental results of Euclidean geometry, the typical high school geometry, concerns the sum of the interior angles in a triangle.



1. Define, in your own words, what an interior angle of a triangle is.

2. What is the sum of the interior angles in a triangle? State this result as a theorem. We will refer to this theorem as the **Triangle Sum Theorem**.

3. Do you have an intuitive understanding of why this result holds? If so, explain in detail. If not, is it surprising to you that every triangle has the same interior angle sum? Explain.
Our goal below will be to understand/prove why this result holds.

Measurement

A protractor is the typical tool used to measure angles.

4. Carefully draw a triangle.

5. Measure each of interior angles in your triangle. What is their sum?

6. Compare your sum with those of your peers. Do your answers agree? If not, try to understand why they do not agree. If so, does this prove the Triangle Sum Theorem? Explain.

7. Cut out your triangle. Now tear off the corners of your triangle. Is there a way you can arrange these three pieces to determine the sum of the angles in the original triangle? Explain.

8. Compare your sum with those of your peers. Do your answers agree? If not, try to understand why they do not agree. If so, does this prove the Triangle Sum Theorem? Explain.

9. Carefully draw another triangle, using a significant portion of a piece of paper. Carefully cut out your triangle. Make a single fold in your triangle so i) one corner of your triangle is folded over so it lies directly on the opposite side, and, ii) the fold line is parallel to the side the corner has been folded on to in i).

10. Can you find two more folds that enable you to determine the sum of the angles in your triangle? Draw a picture which illustrates your construction.

11. How does your origami-like construction compare to those of your peers? Do your results prove the Triangle Sum Theorem?

Myopia

12. Carefully draw another triangle.

13. At one vertex of the triangle continue one of the edges to form an infinite ray that extends beyond the triangle. You have created the exterior angle of the triangle at that vertex. Highlight and label this angle.

14. Now draw infinite rays that extend each of the other edges to show the other exterior angles of your triangle. (Note: At each vertex you have a choice of which line to extend. Extend the edge so that your figure looks like a pinwheel when complete; i.e. all of the exterior angles are measured in the same direction, either clockwise of counter-clockwise.)

15. At a given vertex how is the interior angle related to the exterior angle?

16. Carefully redraw your triangle and its extended edges as if you were viewing it from farther away.

17. Repeat Investigation 16if you viewed your figure from even farther away.

18. Repeat Investigation 16if you viewed your figure from even farther away.

19. Carefully redraw your figure as if you were standing infinitely far away.

20. Do these images suggest a value for the sum of the exterior angles of your triangle? Use your result in Investigation 19 to prove the Triangle Sum Theorem.

Geometric Proofs

To complete a two column proof for geometry theorems, you will use Properties of Congruence, in addition to the Properties of Equality.

Summary	Properties of Congruence
Reflexive Property	$\overline{AB} \cong \overline{AB}$ $\angle A \cong \angle A$
Symmetric Property	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
Transitive Property	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

21. Support each conclusion with a reason.

a. Given: $\overline{AB} = \overline{XY}$
Conclusion: $\overline{XY} = \overline{AB}$ Reason:____________________________

b. Given: $\overline{UP} = \overline{AT}$ and $\overline{AT} = \overline{IN}$
Conclusion: $\overline{UP} = \overline{IN}$ Reason:____________________________

c. Statement: $m \measuredangle 1 = m \measuredangle 1$ Reason:____________________________

22. False Proofs: One of the reasons that justifying each step in a proof is important is because it allows you to find mistakes or inconsistencies in complicated chains of reasoning. For example, here is a false proof about line segments. See if you can find the mistake.

Given: with midpoint C
Prove: AB = 0

Statement	Reason
1. $\triangle ABC$	1. Given
2. $9x = 36$	2.
3. $x = 4$	3.

12. Fill in the reasons that justify each statement to complete the following two column proof:

Statement	Reason
1. $2(x - 12)$ =40	1. Given
2. $2x - 24 = 40$	2.
3. $2x = 64$	3.
4. $x = 32$	4.