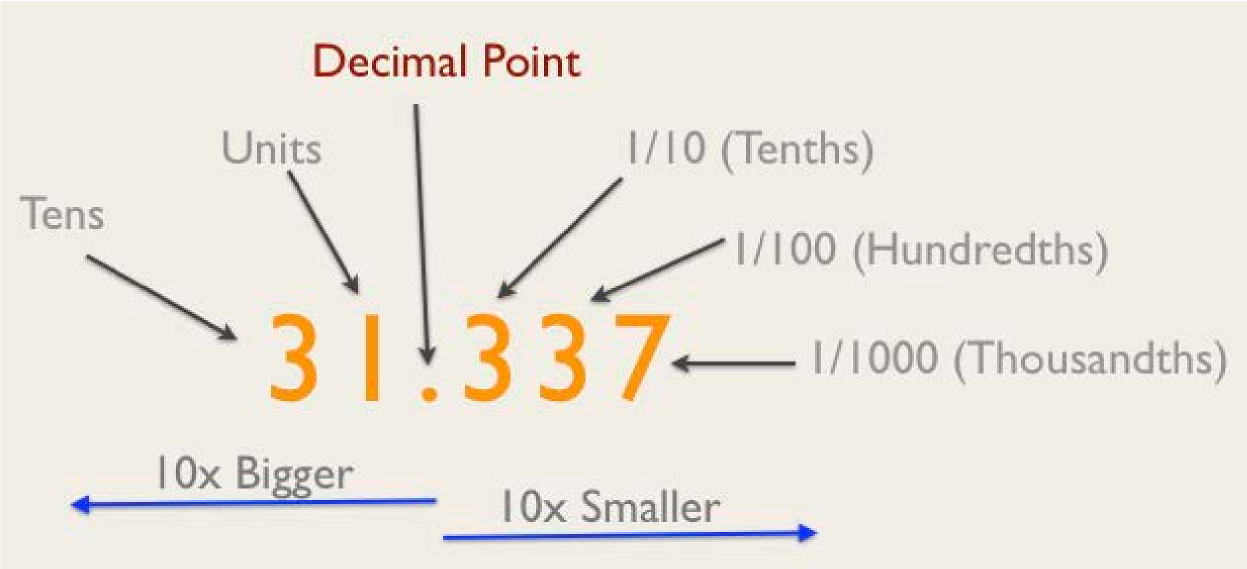
Decimal Number System



In this lesson, you will explore the familiar decimal number sys-tem, and familiarize yourself with the terminology employed in the other number systems. Yes, there are other numbers systems besides binary and decimal!

Let's begin with an introduction to the term **base**. The base of any number system is the number of distinct characters employed for *numeration* when using that system.

For instance, in our common decimal system, there are 10 symbols we use for numeration: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. All numbers within the decimal system are constructed using those characters. The number of available characters is given as the **base of the system** and hence the decimal system is also referred to as the **base-10 number system**.

Other number systems use a different set of characters that may be a smaller or larger set than that used in the decimal system. The binary, or **base-2 system**, employs only the digits 0 and 1, and therefore the base of that system is 2.

1. How many distinct characters are used in the base-3 (or Ternary) number system?

2. List all of the distinct characters used in the base-8 (Octal) number system.

3. List all of the distinct characters used in the base-5 (Quinary) number system.

The quinary number system "is very old, but in pure form it seems to be used at present only by speakers of *Saraveca*, a South American Arawakan language."[1] Many people still use an impure form of quinary numbers - tally marks!

1	|	6	𝍸 |
2	||	7	𝍸 ||
3	|||	8	𝍸 |||
4	||||	9	𝍸 ||||
5	𝍸	10	𝍸𝍸

4. What number is represented by the following?

What about bases larger than 10? Recall that the base establishes the number of distinct characters used, so a true base-11 number system[2], we can't use the symbol 11 to represent the number eleven, because 11 is just two 1s placed next to each other.

5. Create your own symbols to represent the additional characters needed for a base-12 (Duodecimal) number system.

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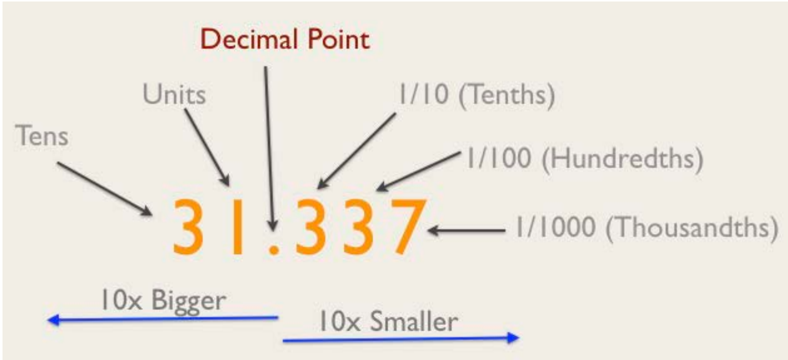
[1] *Quinary Number System*, Encyclopaedia Britannica, http://www.britannica.com/topic/quinary-number-system

[2] The **undecimal** number system was jokingly proposed during the French Revolution to settle a dispute between those proposing a shift to duodecimal and those who were content with decimal



Positional Notation

When a base-10 digit is used in combination or series with other such digits, producing multi-digit numbers such as 318, we recognize that each digit occupies a particular place within the number and that it has a unique place value, capturing its own meaning. The place value is determined by the location of the digit relative to the "decimal point:' Here we must be a bit careful, for in different countries different symbols may be used to mark the separation between integral place values of a number and fractional values. In the United States, for instance, a dot is used to represent the decimal point, while in Great Britain a comma is employed. It might be more accurate to refer to the decimal point as a decimal separator, to avoid confusion, and we will generally adopt that proce-dure here. That being said, the term "decimal point" is widely in use, and hence that term will be taken to be synonymous with "decimal separator."

The decimal separator indicates the line of demarcation between the inte- gral component of the decimal number and the fractional component. To the left of the decimal separator we find the integral component of a number, while to the right we find the fractional component. You are likely to be familiar with the place names for the decimal number system, but we will provide an example as a reminder.

Consider the decimal number 174.3609. The 1 at the start of the number is said to be in the "hundreds" place, the 7 following it to be in the "tens" place, and the 4 following that to be in the "ones"place. Digits to the left of the decimal separator have their place names ending in "s" for all locations. To the right of the decimal separator, the 3 is said to be in the "tenths" place, the 6 following it to be in the "hundredths" place, the 0 following that to be in the "thousandths" place, and the 9 that terminates the number to be in the "ten-thousandths" place. Note that digits to the right of the decimal separator have their place names ending in "ths:'

From the place name of the digit, we obtain what is referred to as its **place value**: For instance, in the previously mentioned example, 174.3609, the l, which occupies the hundreds place, possesses place value 100. The 7 has place value 70 and so on. The digits to the right of the decimal separator have place values such as (for the 6, as an illustration) six hundredths. Using this notion of place values, we can generate the expanded form of a decimal number. This exploded form decomposes the original number into a sum of terms consisting of the individual digits within the number multiplied against an appropriate power of 10. In base-10, you are likely to find the method to be nearly self-evident, but the process may be slightly less natural when we turn our attention to the other bases we will use, and so we'll risk a bit of tedium at this point in order to lay the groundwork for the analogous structure in less familiar bases.

Prior to demonstrating the process, consider an illustration of addition of decimal numbers:

$$300 + 80 + 2 + 0.1 + 0.09 = 382.19$$

The addition is fairly straightforward, but we will now view it in a rather uncon-ventional manner. The symmetric property of equality tells us that the equa-tion can be reversed, yielding

$$382.19 = 300 + 80 + 2 + 0.1 + 0.09$$

The expression now to the right of the equals sign forms the basis for the ex-panded form we are intending. Note that the term "expanded" merely refers, in this case, to a horizontal enlargement of the expression .

Each of the terms on the right side of the equation can be expressed as a single digit multiplied by a power of 10, in a manner evocative of *scientific no-tation*. That is,

$$\begin{aligned} 382.19 &= 300 + 80 + 2 + 0.1 + 0.09 \\ &= 3 \text{ X } 10^2 + 8 \text{ X } 10^1 + 2 \text{ X } 10^0 + 1 \text{ X } 10^{-1} + 9 \text{ X } 10^{-2} \end{aligned}$$

This final form is what we refer to as the **expanded form**. The entire number is written as a sum of terms, each of which uses a digit from the numeration system times *a power of the base* (which is 10 for the decimal number system). Returning to 174.3609, we can decompose the number into the following sum:

$$100 + 70 + 4 + 0.3 + 0.06 + 0.000 + 0.0009$$

Then, proceeding as was done before, we can express the individual terms as a digit multiplied by a power of 10, obtaining

$$1 \text{ X } 10^2 + 7 \text{ X } 10^1 + 4 \text{ X } 10^0 + 3 \text{ X } 10^{-1} + 6 \text{ X } 10^{-2} + 0 \text{ X } 10^{-3} + 9 \text{ X } 10^{-4}$$

You may be wondering if it is necessary to show the term where the digit is zero. In the fully expanded form of the decimal number, all the digits from the original number should be shown for the sake of completeness and systematization. Thus, it would technically be a mistake to suppress that term from the expansion.

In each of the following, identify the place name and place value of every digit.

6. 1,387

7. 281.793

8. 0.01379

9. 10,002.00208

In each of the following, give the expanded form of the decimal number.
10. 16,039.177

11. 201.9938

12. 123,654.92846

13. 874.983

For the following, give the decimal number whose expanded form is shown.
14. $3 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0$

15. $0 \times 10^0 + 1 \times 10^{-1} + 3 \times 10^{-2}$

16. $3 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-2} + 1 \times 10^{-3} + 5 \times 10^{-4} + 9 \times 10^{-5}$

17. $2 \times 10^4 + 9 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 5 \times 10^0 + 3 \times 10^{-1} +$
7×10^{-2}

Significant Digits

We close out this section with a final bit of terminology: in a number repre-sented in the base-10 system, the digit farthest to the left is referred to as the most significant digit (MSD), and the digit farthest to the right is referred to as the least significant digit (LSD). In the following base-10 number, the MSD is 5, while the LSD is 4:

52.374

Referencing the problems indicated, for each of the following, identify the LSD and MSD.
18. Problem 6
19. Problem 7
20. Problem 8
21. Problem 9
22. Problem 10
23. Problem 11

24. Problem 12

Give examples of decimal numbers satisfying the following conditions (answers will vary).

25. A whole number having MSD is 5 and LSD is 8.

26. A number having Os in the tens, hundredths, and thousands places.

*	1	2	3	4	10	11	12	13	14	20
1	1	2	3	4	10	11	12	13	14	20
2	2	4	11	13	20	22	24	31	33	40
3	3	11	14	22	30	33	41	44	102	110
4	4	13	22	31	40	44	103	112	121	130
10	10	20	30	40	100	110	120	130	140	200
11	11	22	33	44	110	121	132	143	204	220
12	12	24	41	103	120	132	144	211	223	240
13	13	31	44	112	130	143	211	224	242	310
14	14	33	102	121	140	204	223	242	311	330
20	20	40	110	130	200	220	240	310	330	400

Quinary Multiplication Table

https://en.wikipedia.org/wiki/List_of_numeral_systems

https://en.wikipedia.org/wiki/Ancient_Egyptian_multiplication

http://atozteacherstuff.com/pages/296.shtml

http://www.storyofmathematics.com/sumerian.html

https://www.youtube.com/watch?v=FfXmnzaDav8

http://csunplugged.org/binary-numbers/#Downloads

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