Rubik’s Cube

> The Cube is an imitation of life itself –or even an improvement on life. The problems of puzzles are very near the problems of life, our whole life is solving puzzles. If you are hungry, you have to find something to eat. But everyday problems are very mixed–they’re not clear. The Cube’s problem depends just on you. You can solve it independently. But to find happiness in life, you’re not independent. That’s the only big difference.
>
> **Erno˝ Rubik** (Architect; 1944 -)

1.1 History and background

The Rubik’s Cube was invented by Hungarian architect Ern˝o Rubik in 1974 (see Figure 1.1). His original motivation for developing the cube was actually a curiosity about architectural configurations in space.

> I love playing, I admit it, I particularly love games where the partner, the real opponent is nature itself, with its really particular but decipherable mysteries. The most exciting game for me is the space game, the search of possible space shapes, that is to say the logical and concrete building of various layouts.

Known as the ”Magic Cube,” the Rubik’s Cube became a hit in Hungary starting in 1977. The other person essential in making the Rubik’s Cube famous was hobby mathematician and businessman Tibor Laczi. He was instrumental in bringing the cube to the west in the early 1979’s. As a result, Rubik became the first self-made millionaire in the Communist block. As of January 2009, about 350 million Rubik’s Cubes have been sold worldwide.

1.1.1 Random Moves

> Space always intrigued me, with its incredibly rich possibilities, space alteration by (architectural) objects, objects’ transformation in space (sculpture, design), movement in space and in time, their correlation, their repercussion on mankind, the relation between man and space, the object and time. I think the CUBE arose from this interest, from this search for expression and for this always more increased acuteness of these thoughts …

For those tempted to just keep making random moves in the hope of solving the cube, there are 43,252,003,274,489,856,000 different configurations of the cube, which is approximately forty-three quintillion.

Example Investigation: *Find a way to make concrete and visible how large a variety of forty-three quintillon different cube configurations are, and what that might mean for solving the cube by making random moves.*

As an illustration how to think about such an investigation, we include a few ways of reasoning and documenting our thinking right here.

next to each other in a long line, for example. How long do you think that line would be? Take a wild guess! Once around the island of Manhattan? From New York to San Francisco? Once around the globe? All the way to the moon? All the way to the sun? All the way to the next star? All the way to the next galaxy? All the way across the universe?

We could also wonder how long it would take us to cycle through all these different configurations if we could make one move with our cube every second (which is reasonable but doesn't give us much time for thinking). Take a wild guess!

Now, let us take a closer look together: Given that a cube is about 57 millimeters wide, the line of cubes would stretch for 261 light years[1]. For comparison, the sun is about 8 light minutes away, and the closest star to earth is about four light-years away[2]. Our own galaxy, the Milky Way, is about 100,000 light years across. The closest galaxy similar in size to our own, the Andromeda Galaxy, is about 2 million light years away. So the line of cubes would stay within our own galaxy, but extend far beyond the closest star. In fact, Spica–the brightest star in Virgo–is about 260 light years away; see below.

We can look at this question in some other way: If it takes you one second to make one random move, and you manage to get a new configuration after every move, then it would take you at least 1.37 trillion years to make all the different configurations (at 31,557,600 seconds in a year). Scientists estimate the age of the universe is about 13.75 billion years (give or take about 170

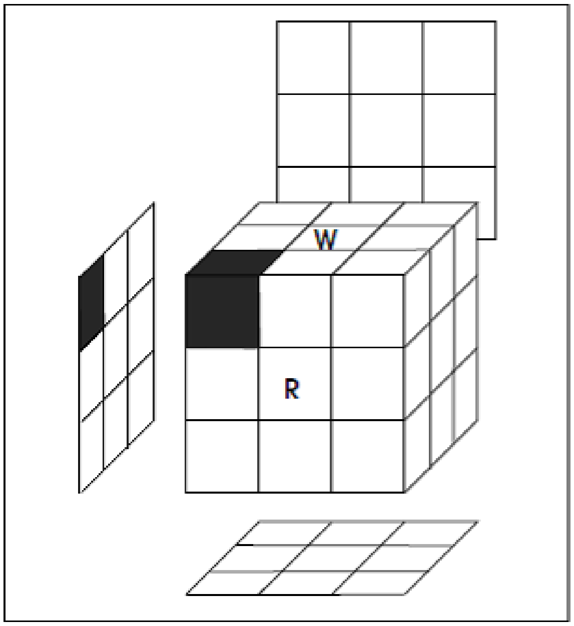
[1] One light year: 9,460,730,472,580.8 km. Width of the cube: 57 mm. [2]Proxima Centauri

million years). Since 1.37 trillion is about 1370 billion years, it would take one hundred times the age of the universe to cycle through all possible configurations of the Rubik's Cube in this way. You may be lucky if the solved cube comes early—or you may end up twisting it in vain through the end of the universe.

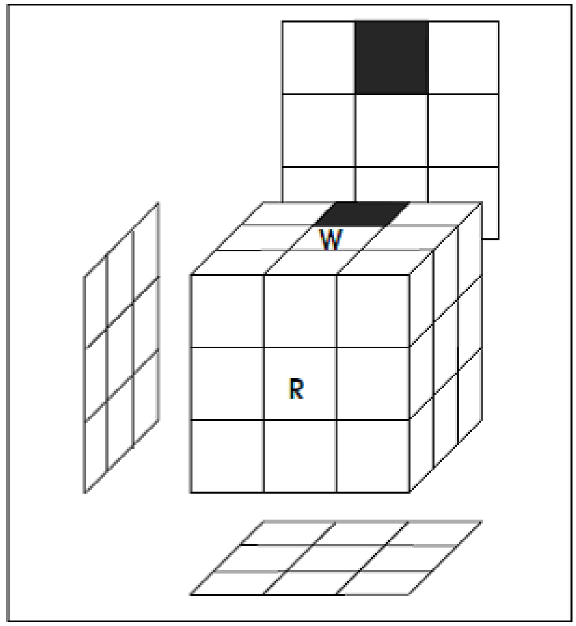
Getting to Know the Cube

The Rubik's Cube consists of many little *cubies*. The standard cube has six different colors: white, red, blue, green, orange, and yellow. The images we use have the following configuration: when completely solved, green is opposite yellow, orange is opposite red, white is opposite blue; as in http://www.schubart.net/rc/. If your cube looks different, don't worry!

1. How many cubies does the Cube have?
2. How many different stickers does each cubie have? Do they all have the same number of colored stickers? Explain in detail.
3. Pick a corner cubie. Can you move the cubie so that it's not in a corner any more? Explain your observations.
4. Pick a cubie that is not at a corner. Can you move this cubie into a corner position?
5. Turn your cube so that the white *center cubie* is facing up and the red center cubie is in front, facing you, as in Figure 1.3(a). Notice the marked corner cubie in the upper front left corner (the face that would otherwise be invisible on the left hand side is marked on the separately drawn; imagine a mirror drawn in this location. Clearly mark in the Figure below all the different positions that you can get this corner cubie into, while keeping the white center cubie on top and the red center cubie in front. Explain any patterns you see.



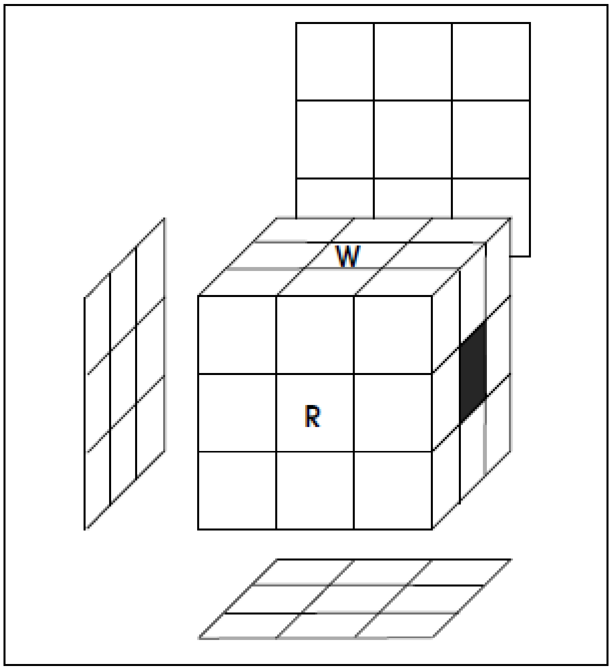
(a) Where can the shaded corner cubie go, while keeping the white and red centers in place?



(b) Where can the shaded edge cubie go?

6. Similarly, consider the marked *edge cubie* in Figure 1.3(b). Clearly mark in Figure 1.3(b) all the different positions that you can get this edge cubie into, while keeping the white center cubie on top and the red center cubie in front. Explain any patterns you see.

Finally, consider the marked center cubie in Figure 1.4. Clearly mark in Figure 1.4 all the different positions that you can get this center cubie into, while keeping the white center cubie on top and the red center cubie in front. Explain any patterns you see.



Summarize your findings and your reasoning in a few paragraphs so that you could present them to the whole class.

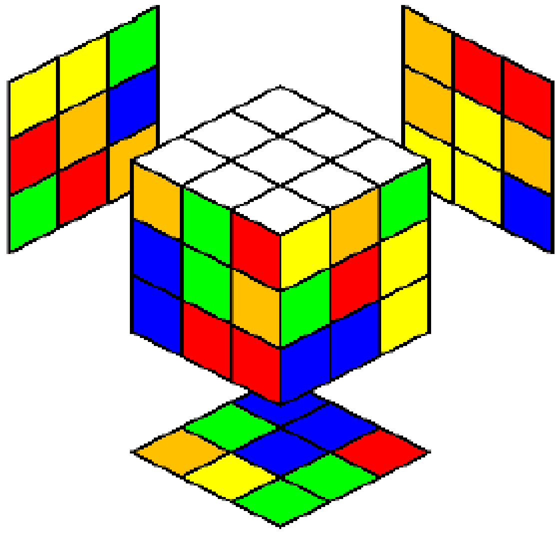
Solving One Face

The time needed for the following Independent Investigations may be measured in days, not in minutes.

Independent Investigation: Solve the white face on your own. It may be easiest to make white the top face, so you see what is going on. Take your time to get to know your cube. Observe which moves affect which cubies (and what cubies are left unaffected.) You can do it.

Once you have one face all white, congratulations! The nine cubies that all have one white sticker form the top *layer*. Thus, a layer consists of cubies, while a face consists of stickers. Now look at the color of the stickers around the edge of the white layer cubies. Do the three cubies next to each other on each face have the same color or different colors?

Consider, for example, Figure 1.5.



Solving the first layer

9. In Figure 1.5, we see a cube with a white side on top. Consider the four upper edge cubies (those with white stickers on top). Are they already in the correct position? How do you know? If not, describe where they would need to go in the completely solved cube (while keeping the white center cubie on top and the green center cubie in front).

10. Now, consider the four upper **corner** cubies. Are they already in the correct position? How do you know? If not, describe where they would need to go in the completely solved cube (while keeping the white center cubie on top and the green center cubie in front).

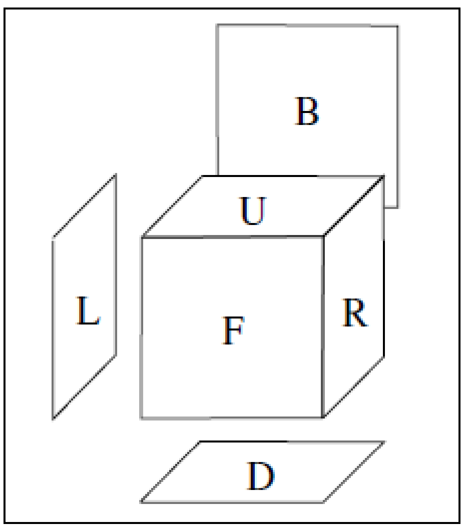
11. **Classroom Discussion:** You may have noticed that it is difficult to precisely describe your strategies. What are effective ways we can use to clearly describe individual cubies, in words or in symbols? Explain why the methods you propose are clear and effective.

Independent Investigation: Rearrange the top layer cubies so that the stickers along the edge on each face all have the same color. In the process, find your own methods for moving particular cubies in a way that leaves other cubies unaffected.

Once you know how to solve the white top layer, move on to next set of investigations, starting with Investigation **14**. They ask you to describe your moves for bringing certain cubies into a new position, while keeping the rest of the white top layer unaffected (i.e. once your moves are done, each of those cubies needs to be back in the place where it started out). We don't care at this point whether cubies in the second or third layer change position. It's OK if they end up in a different place. Find a way to describe your moves clearly enough so that somebody else could use these as instructions without any further aid.

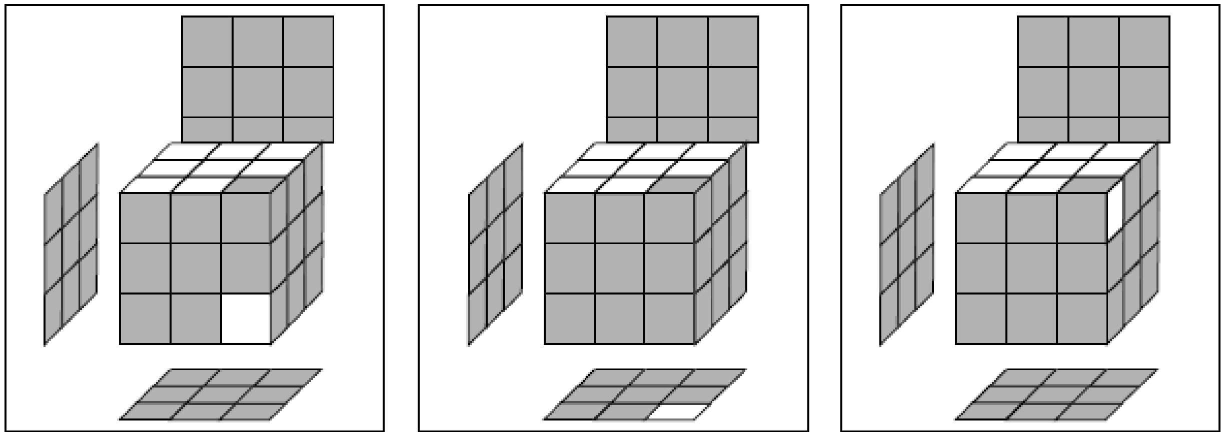
1.3 Moving Specific Cubies in Specific Ways

The conventional notation for the different faces, usually named **Singmaster notation**, is shown below: **U**p, **D**own, **F**ront, **B**ack, **L**eft, and **R**ight. Individual cubies are referred to using lowercase letters: for example, the corner cubie in the right, down, and back layer is called *rdb*, for short.



14. In Figure 1.7(a), complete the white top face; i.e. move the *drf* corner cubie into the *urf* position so that the white sticker faces up. Leave the rest of top layer unaffected.
15. **Classroom Discussion:** You may have noticed that it is hard to precisely describe your moves. What are effective ways we can use to clearly describe cube **moves**, in graphical representations, in words or in symbols? Why are the methods you propose clear and effective?

The conventional notation for denoting cube moves uses the abbreviations for the faces, as seen in Figure 1.6. We can turn a face clockwise as we look at it (denoted as U ,D ,F ,B ,R ,L) or in a counter-clockwise direction (denoted as U^{-1},D^{-1},F^{-1},B^{-1},R^{-1},L^{-1}). Notice that U^{-1} undoes precisely what the move U does; because of this, we call U^{-1} "*U inverse*."



(a) Move the drf corner. (b) Move the drf corner. (c) Fix the urf corner.

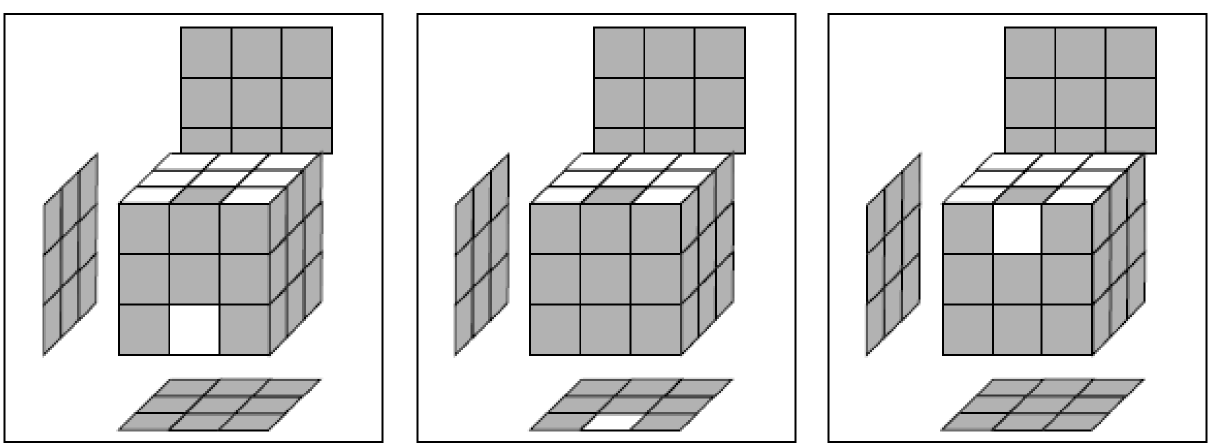
16. Complete the white top face in Figure 1.7(b) by moving the *drf* corner. Clearly describe your moves.

17. Complete the white top face in Figure 1.7(c) by fixing the *urf* corner. Clearly describe your moves.

18. Complete the white top face in Figure 1.8(a). Clearly describe your moves.

19. Complete the white top face in Figure 1.8(b). Clearly describe your moves.

20. Complete the white top face in Figure 1.8(c). Clearly describe your moves.



(a) Move the df edge. (b) Move the df edge. (c) Fix the uf edge.