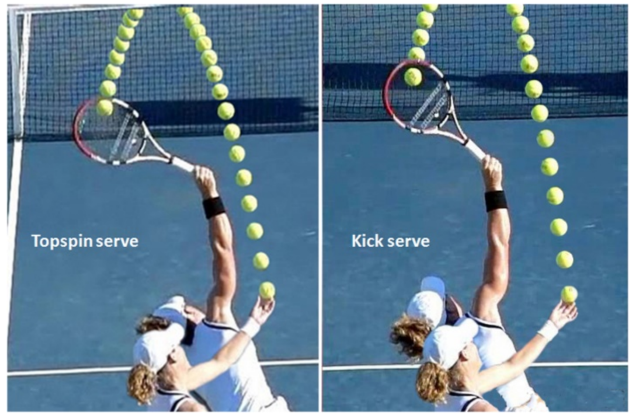
2.10 Ad-Out



Instantaneous Velocity

This brief lesson will serve as a return to the problem of falling (and tossed objects). This time you will use your hard-earned knowledge of limits to evaluate the instantaneous velocity of the object.

Suppose that you have a moving object whose position at time t is given by a function s. You know that the average velocity of the object on the time interval $[a, b]$ is $AV_{[a,b]} = \frac{s(b)-s(a)}{b-a}$.

One defines the ***instantaneous velocity*** at a to be the limit of average velocity as b approaches a.

Note particularly that as $b \to a$, the length of the time interval gets shorter and shorter (while always including a). In the next lesson, you will introduce a helpful shorthand notation to represent the instantaneous rate of change. For now, write $IV_{t=a}$ for the instantaneous velocity at $t = a$, and thus

$$IV_{t=a} = \lim_{b \to a} AV_{[a,b]} = \lim_{b \to a} \frac{s(b) - s(a)}{b - a}$$

Equivalently, if you think of the changing value b as being of the form $b = a + h$, where h is some small number, then You may instead write

$$IV_{t=a} = \lim_{h \to 0} AV_{[a,a+h]} = \lim_{h \to 0} \frac{s(a + h) - s(a)}{h}$$

Again, the most important idea here is that to compute instantaneous velocity, you take a limit of average velocities as the time interval shrinks. Two different activities offer the opportunity to investigate these ideas and the role of limits further.

Investigation1: Consider a moving object whose position function is given by $s(t) = t^2$, where s is measured in meters and t is measured in minutes.

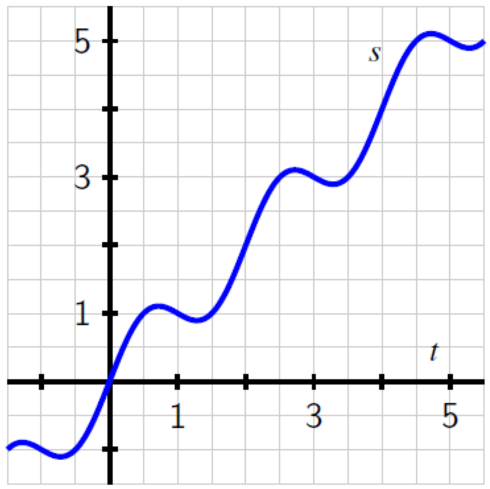
1a) Determine the most simplified expression for the average velocity of the object on the interval $[3, 3 + \mathrm{h}]$, where $\mathrm{h} > 0$.

b) Determine the average velocity of the object on the interval [3, 3.2]. Include units on your answer.

c) Determine the instantaneous velocity of the object when $\mathrm{t} = 3$. Include units on your answer.

The last investigation of this section asks you to make some connections among average velocity, instantaneous velocity, and slopes of certain lines.

Investigation2: For the moving object whose position s at time t is given by the graph below, answer each of the following questions. Assume that s is measured in feet and t is measured in seconds.



2a) Use the graph to estimate the average velocity of the object on each of the following intervals: [0.5, 1], [1.5, 2.5], [0, 5]. Draw each line whose slope represents the average velocity you seek.

b) How could you use average velocities or slopes of lines to estimate the instantaneous velocity of the object at a fixed time?

c) Use the graph to estimate the instantaneous velocity of the object when $t = 2$. Should this instantaneous velocity at $t = 2$ be greater or less than the average velocity on [1.5, 2.5] that you computed in (a)? Why?

II. Practice

1. A bungee jumper dives from a tower at time t = 0. Her height h in feet at time t in seconds is given by $h = 100t) \cdot e^{-0.2t} + 100$.

   a) Write an expression for the average velocity of the bungee jumper on the interval [1, 1 + h].

   b) Use computing technology to estimate the value of the limit as $h \to 0$ of the quantity you found in (a).

   c) What is the meaning of the value of the limit in (b)? What are its units?

2. A pumpkin is dropped from the top of a 100 meter tower. Its height above ground after t sec is $100 - 49t^2$ m. What is the instantaneous velocity of the pumpkin 2 sec after it is dropped? (Be sure to include the appropriate unit.)

3. At t sec after lift-off, the height of a rocket is $3t^2$ ft. What is the instantaneous velocity of the rocket after 10 sec?

4. The height of a rock dropped from a 200-foot cliff on Mars is $h = 200 - 1.86t^2$ meters with t in seconds. Find the velocity of the rock at $t = 1$ sec.

5. The height of a calculator dropped from a 500-foot cliff on Jupiter is $h = 500 - 11.44t^2$ feet with t in seconds. Find the velocity of the rock at $t = 1$ sec.