Solving the Cube: Corners First Style

There are many different approaches to solving the cube. The main goal here is not just to solve one particular aspect of the cube **once** (e.g. having the top face show all white), but for you to develop reliable methods that will **always** allow you to solve certain aspects of the cube. These methods are related to the magical moves in the preceding lesson.

Where do these magical moves come from? They come from great insight, research, deep geometric ideas, as well as insights from modern abstract algebra. They are not obvious. They may not seem natural. They are certainly not something that one could come up with easily. They are magical not just in what they do, but that they have been discovered at all.

There are several of these magical moves. Each will take a bit of time to understand and get used to. Once you have reached a particular goal, celebrate excitedly for a minute, congratulate yourself for excellent work, take careful notes on any important observations you made, draw a picture, take a photo, then mess up the cube and do it all over again. Once you've really got this move down, move on to the next.

This solution method is based on the book *Adventures in Group Theory* by David Joyner.
As before, the color of the center cubies tells us what color this face will be in the solved cube. Because of this fact, you can determine precisely which cubies should go where.

You may use moves from above. But be careful since they may scramble up parts of the cube that you have already solved.

The following method has the following main strategy:

- First bring all of the corner cubies into the correct position.
- Next bring all of the edge cubies into the correct position.
- Fix the orientations of the corner cubies.
- End by fixing the orientation of the edge cubies.

With this approach the cube may continue to look fairly messy for a fairly long time. It may be helpful for two people to work together on solving the cube this way.

Fixing the Positions of the Corner Cubies

Two tools - two new magical moves - are required to fix the eight corner cubies. We will call the first move CR for **corner rotation**. It is:

$$CR = U^{-1} R^{-1} D^{-1} R U R^{-1} D R$$

This move effects exactly three cubies: ulf, urf, and drf.

1. By doing CR a number of times, determine the impact of this move on the ulf, urf, and drf cubies.

2. Draw arrows on the cube in the Figure below to show where move CR moves cubies.
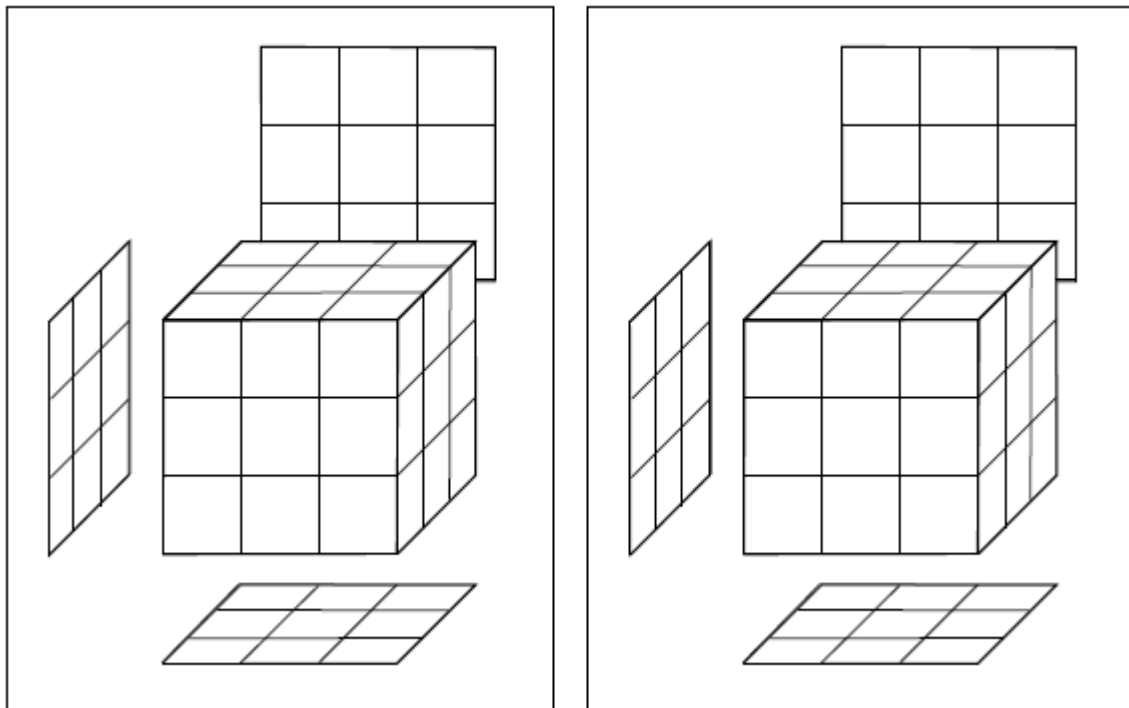
3. What is the order of the move CR?

We will call the next move CS for **corner swap**. It is:

$$CS = U\ F\ R U\ R^{-1}\ U^{-1}\ R U\ R^{-1}\ U^{-1}\ R U\ R^{-1}\ U^{-1}\ F^{-1}$$

This move effects exactly eight up cubies - all of the up cubies but the center. No other cubies are effected by this move.

4. By doing *CS* a number of times, determine the impact of this move on the *ubr* and *ufl* cubies.

5. By doing *CS* a number of times, determine the impact of this move on the *ubl* and *ufr* cubies.

6. By doing *CS* a number of times, determine the impact of this move on the *uf,ul,ub,* and *ur* cubies.

7. Draw arrows on the cube in the Figure below to show where move *CS* moves cubies.
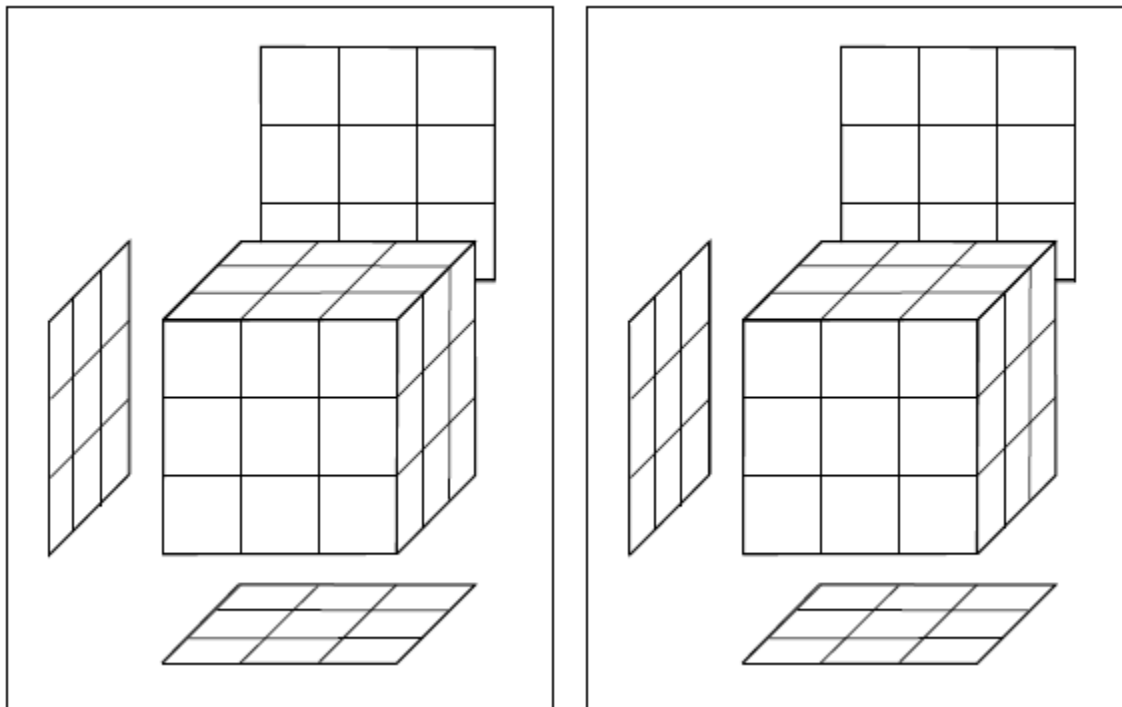
8. What is the order of the move *CS*?

9. Solve a layer of your cube - a face with the adjoining row of cubies around the edges correct as well.

10. Now use the two moves you have learned in this section to try to fix the position of the remaining four corners of your cube without messing up your solved face. Were you able to? If so, celebrate! Then mix your cube up and do it again. If not, see if you can get some help from a peer.

11. Classroom Discussion: Come together with your peers and discuss the use of the moves CS and CR. Were each of you able to fix the four remaining corners? If some were not, can others help? If each cube's corners can be solved, can you explain how you know these two moves suffice? Alternatively, is there an example where you can describe precisely how these moves do not suffice?
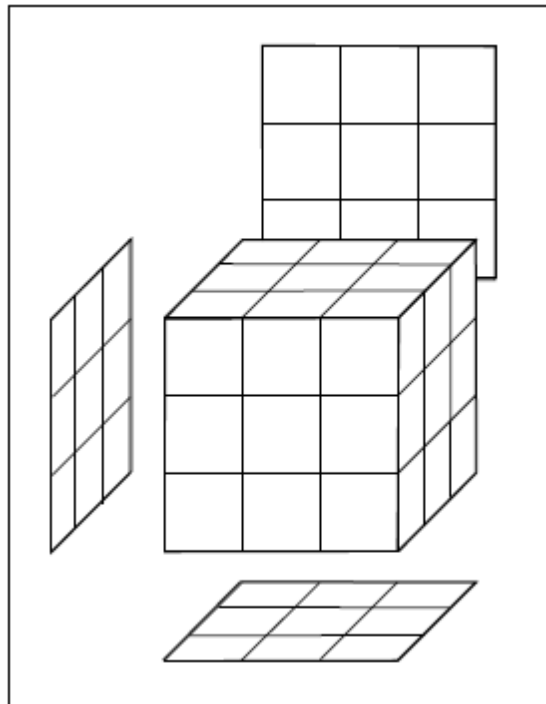
Fixing the Positions of the Edge Cubies

Another move that is an important tool, which we call ER for **Edge Rotation**, is:

$$ER = M_R^2 \, U^{-1} \, M_R^{-1} \, U^2 \, M_R \, U^{-1} \, M_R^2.$$

This move effects exactly three cubies: *uf,ul,ur.*

12. By doing *ER* a number of times, determine the impact of this move on the cube.

13. Draw arrows on the cube in Figure 1.12 to show where move ER moves cubies.

14. What is the order of the move *ER*?

15. Can you use this move to fix the position of *all* of the remaining edge cubies of your cube. Were you able to? If so, celebrate! Then mix your cube up and do it again. If not, see if you can get some help from a peer.

16. **Classroom Discussion:** Come together with your peers and discuss the use of the moves ER. Were each of you able to fix the edges? If some were not, can others help? If each cube's edges can be solved, can you explain how you know this move suffices? Alternatively, is there an example where you can describe precisely how this move does not suffice?

Fixing the Orientations

Now that you have all of the cubies in place, it is likely that many of them will not be oriented correctly. You've already seen that that move M_2 simply changes the orientation of cubes. You'll need this move in addition to two others that will help orient some of the other cubies that need to be reoriented.

We call the first CO for **Corner Orientation**. It is:
$$CO = (R^{-1} D^2 R B^{-1} U^2 B)^2$$

This move effects exactly two cubies: *ufr* and *dbl*.

17. By doing *CO* a number of times, determine the impact of this move on the cube.

18. Draw arrows on the cube to show how CO reorients cubies.

19. What is the order of the move *CO*?

20. Can you use this move to reorient *all* of the corner cubies of your cube? If so, celebrate! Then mix your cube up and do it again. If not, see if you can get some help from a peer.

We call the second EO for **Edge Orientation**. It is:
$$EO = (M_R U)^3 U (M_R^{-1} U)^3 U$$

This move effects exactly four cubies: *uf* and *ub.*

21. By doing *EO* a number of times, determine the impact of this move on the *ub,ul,df*, and *db* cubies.

22. Draw arrows on the cube in the Figure below to show how *EO* reorients cubies.

23. What is the order of the move *EO*?

24. Use *EO* and M_2 to try to fix the position of the remaining four corners of your cube. These moves will suffice - you can convince yourself later. Get help from peers if needed. Then
...*Celebrate*!!

25. Classroom Discussion: What were some of the challenges you encountered in working only with the limited tools given in the Investigations? How did you manage to overcome them?