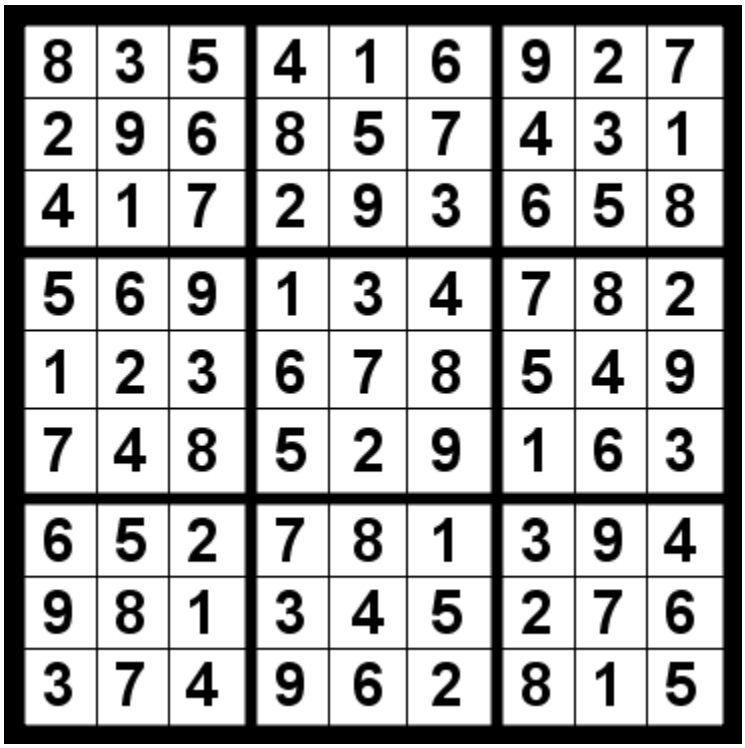

8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
3	7	4	9	6	2	8	1	5

Sudoku

Regular Sudoku is a single-player puzzle consisting of a square 9×9 grid usually containing digits ranging from 1 through 9. A significant number of grid entries are empty. We call each of the digits that are given a **clue**. See Figures 2.1(a)-2.2(b) for a few example Sudoku puzzles. The large square is subdivided into nine 3 × 3 sub-squares. The goal is to fill the digits 1 through 9 into the open grid positions according to the following three rules:

- Each **row** must contain each of the digits 1 through 9 exactly once. (Notice that this means that each of those digits must occur somewhere along the row, and none of them can be duplicated.)
- Each **column** must contain each of the digits 1 through 9 exactly once.
- Each of the small 3 × 3 squares must contain each of the digits 1 through 9 exactly once.

Regular Sudoku puzzles are designed to have one unique solution, i.e. there is only one possible correct answer for each space in the grid. If you find yourself guessing between a variety of possible answers, you may guess incorrectly. Make sure you eliminate all but a single answer before filling in a final solution! Sudoku puzzles are widely available in newspapers, books, and online. There is even an iPhone App.

3	2	4			9		7	
		7			2			8
8		6		5		1	2	
9	3		5	1				
			8		4			
				9	3		5	1
	4	3		7		8		9
6			9			4		
	8		6			5	3	7

(a) Entry level.

5					2		4	1
	4	1		8		3	6	
	6	3	9				8	
	5					6		
8		6		7		1		3
		9					7	
	3				8	2	9	
	9	5		2		8	1	
7	8		1					6

(b) Entry level.

Before we go any further, take a stab a solving a few puzzles. Figures above and below show four Sudoku puzzles of varying levels of difficulty:

1. Pick two of the puzzles that are somewhat challenging for your level of experience with Sudoku. Solve these two puzzles. (If you are ready for an even more challenging Sudoku puzzles, you may replace these by more challenging ones. Clearly note your source.) As you solve the puzzle, keep an eye on the processes and the strategies you use.

	3		5			7		
		6				9		
	4		9	2	7	1	6	
4		5		9	6			
			2		3			
			7	4		6		1
	5	2	8	7	9		1	
		4				5		
		3			4		9	

(a) Medium.

6			7			9		
								4
	2	9	1					6
		4	5		7			
9		5	4		6	2		3
			9		8	4		
2					1	3	9	
5								
		7			3			1

(b) More challenging.

2. **Classroom Discussion:** Summarize the processes and strategies you use to solve these puzzles and explain why they are helpful to you in solving the Sudoku puzzle.

History

Latin Squares

Sudoku puzzles are an example of what the famous mathematician **Leonhard Euler** (Swiss mathematician and physicist; 1707 - 1783) called *Latin Squares*: these are square tables filled with digits, letters, or symbols so that each of the entries occurs only once in each row or column. (Notice that Sudoku puzzles have the added requirement that each of the little 3 × 3 contain the digits 1–9 only once.) Figure 2.3 shows a beautiful example of a Latin Square which does not use digits or letters: in this stained glass window, each of the seven colors occurs only once in each row and in each column.

3. Working on your own, turn the color Latin Square displayed in the stained-glass window in Figure 2.3 into a Latin Square that uses numbers. Describe your process in detail. (If this printed document is not clear enough to make out the color detail, you will find a color image of the window at http://en.wikipedia.org/wiki/File:Fisher-stainedglass-gonville-caius. jpg.)

4. Now, find a different set of numbers that also fit this particular stained glass window. How do your results compare? Explain your observations.

5. Compare your results with those of other students. In what ways are the different Latin Square that were created the same? In what ways are they different? Explain.

6. How many different ways do you think there are to create a number version of the color Latin Square from Figure 2.3 (say, using the numbers 1 − 7)? Document your process of thinking about this question. Carefully explain your answers and ideas. Clearly explain why your reasoning and results makes sense.

Latin Squares were know long before Euler gave them this name, and in cultures far from the Latin world. You can find examples of Latin squares in Arabic literature over 700 years old. An ancient Chinese legend goes something like this: Some three thousand years ago, a great flood happened in China. In order to calm the vexed river god, the people made an offering to the river Lo, but he could not be appeased (Figure 2.4 shows a Ming dynasty scroll depicting the nymph of the Lo river).

Each time they made an offering, a turtle would appear from the river. One day a boy noticed marks on the back of the turtle that seemed to represent the numbers 1 to 9. The numbers were arranged in such a way that each line added up to 15, as in Table 2.2.1. Hence the people understood that their offering was not the right amount.

7. In the Lo Shu square, describe all the number patterns and relationships you can find.

Figure 2.5 shows one of the best-known depictions of a Latin Square in Western art. It is included in a woodcut called “Melencolia I” by the German artist **Albrecht Du¨rer** (painter, printmaker and theorist; 1471 - 1528), conventionally regarded as the greatest artist of the Northern Renaissance. Figure 2.6 shows ”Melencolia” in its entirety. Du¨rer

spent significant time in Italy and remained in communication with most of the major artists of the time, including **Raphael** (Italian painter; 1483 - 1520) and the proverbial Renaissance man **Leonardo da Vinci** (sculptor, painter, scientist; 1452 - 1519).