8	3	5	4	1	6	9	2	7
2	9	6	8	5	7	4	3	1
4	1	7	2	9	3	6	5	8
5	6	9	1	3	4	7	8	2
1	2	3	6	7	8	5	4	9
7	4	8	5	2	9	1	6	3
6	5	2	7	8	1	3	9	4
9	8	1	3	4	5	2	7	6
3	7	4	9	6	2	8	1	5

Removing Clues

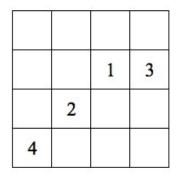
One way to try to make a puzzle more difficult (or at least require more work) is to remove some of the clues.

7			3		9	4	6	
4		1			2		8	9
6	3			8		2		
		4					1	
2	7						9	4
	1					7		
		7		2			3	6
1	6		5			9		7
	9	2	6		3			8

Remove one digit, making sure it still has a unique solution.

- **1. Classroom Discussion:** In the previous puzzle, you can remove a "7" so that it still has a unique solution. Why can you be sure that the puzzle is still unique? Are there any other digits you can safely remove?
- 2. For the Sudoku puzzle below left, find one digit you could safely remove from the puzzle. (Make sure that it still has one unique solution!)

4			
		1	
	2	3	
			4



- (a) Jose's Sudoku puzzle.
- (b) Karen's Sudoku puzzle.
- 3. For the Sudoku puzzle above left, find one digit you could safely remove from the puzzle.
- 4. For the Sudoku puzzle above right, find one digit you could safely remove from the puzzle.
- 5. Jose invented the Sudoku puzzle above left. What do you notice as you solve the puzzle?
- 6. Karen invented the Sudoku puzzle above right. What do you notice as you solve the puzzle?
- 7. It seems that creating Sudoku puzzles is not that easy. How do you think Joe and Karen should have gone about creating such puzzles?
- 8. Create four different puzzles yourself, then exchange them with a partner. Solve the puzzles you receive. Does each have a solution? Is it unique? (The last page of this leson has empty templates for 4×4 Sudoku puzzles.)
- **9. Classroom Discussion:** From among the puzzles you created, pick your favorite one and share it with the class. Comparing all the puzzles, your creation strategies and the solutions, what patterns, similarities, and differences do you notice?
- 10. Independent Investigation: It is an open problem to know what is the smallest number of clues needed for a 9×9 Sudoku puzzle² (with a unique solution). Investigate what is the smallest number of clues needed for a 4×4 puzzle (with a unique solution)?

11. Independent Investigation: What is the **largest** number of clues you can have in a 4×4 puzzle that still has **no unique** solution?

