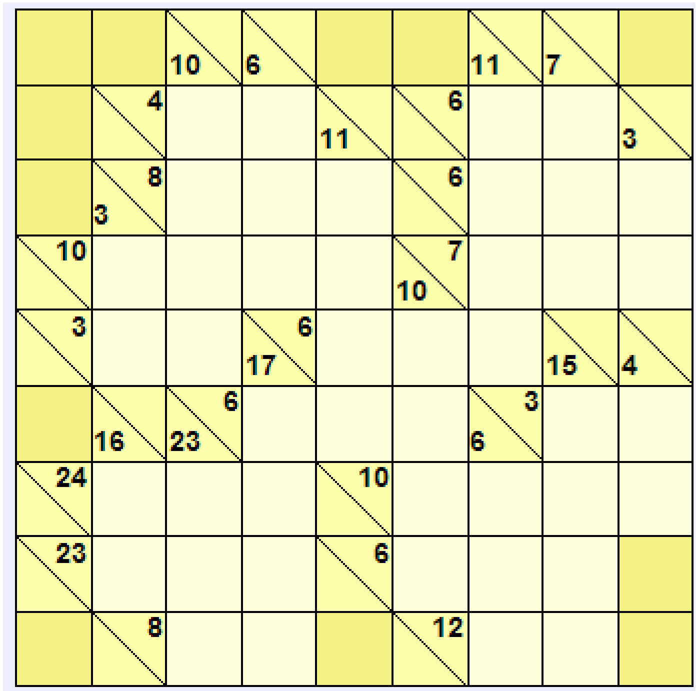


Kakuro

1. What made magic squares magical is that the sums of rows, columns, diagonals, and perhaps other groups of entries all have a common sum. What can you say about the sums of the columns of a Sodoku puzzle? The rows? The diagonals? Other groups of entries?

2. For a Latin square made with numbers, what can you say about the sums of the columns? The rows? The diagonals? Other groups of entries?

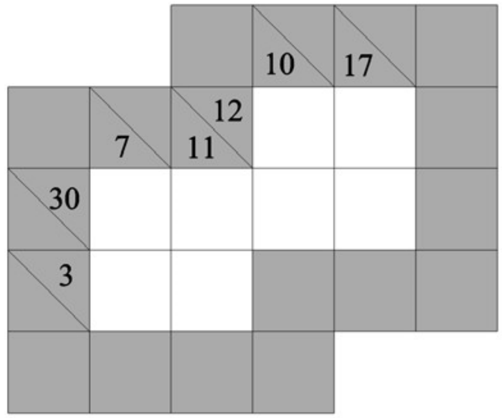
3. Use Investigation **1** to repharse the rules of Sodoku in terms of sums.

Kakuro is a puzzle with a history and popularity that resemble those of Sodoku. **Kakuro** "boards" are grids made of squares, looking much like a typical crossword puzzle board - many blank squares with a number of solid black squares. Taken together, all of the blank squares that are adjacent without interruption by a black square - in a given row or column are called a **run**. Like Sodoku, to solve a Kakuro puzzle you need to fill in all of the blank squares using only the numbers 1 - 9. The differences between the puzzles - other than their shape - are the rules and the clues:

- There is a clue for every run - the clue tells you what the *sum of the terms in the run* must be. No other clues are given; i.e. unlike Sodoku, no squares have been filled in.

- No number can be repeated in any run.

Like Sodoku puzzles, the solution to a Kakuro puzzle is expected to be unique.

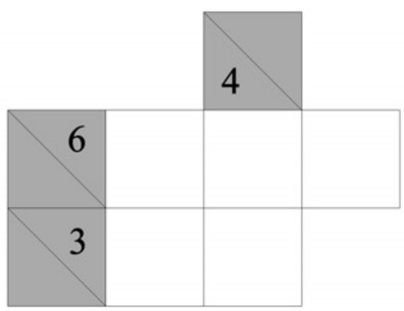


Junior Kakuro puzzle #1.

4. Try to solve the Kakuro puzzle above.

5. What did you find difficult about this puzzle? What did you find easy? What strategies did you use?

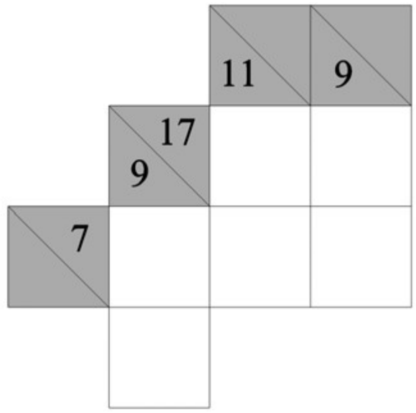
6. In the figure below is a *excerpt* of a Kakuro puzzle. How can you tell that it is only a portion of a puzzle?



Kakuro excerpt #1.

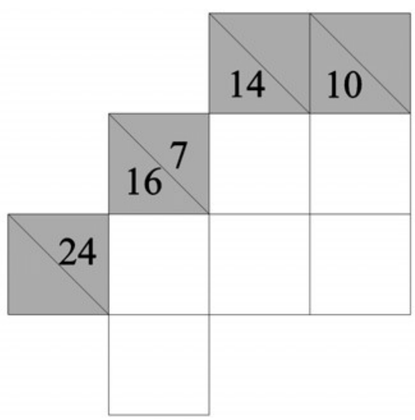
7. Can you solve the excerpt of the puzzle above? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.

8. Can you solve the excerpt of the puzzle in the figure below? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.



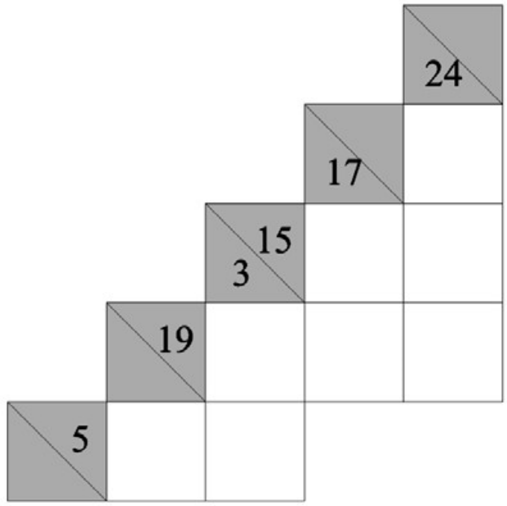
Kakuro excerpt #2.

9. Can you solve the excerpt of the puzzle in the figure below? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.



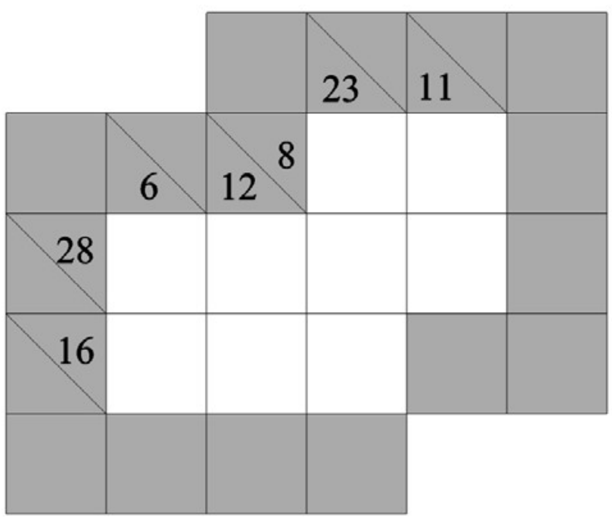
Kakuro excerpt #3.

10. Can you solve the excerpt of the puzzle in the figure below? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.



Kakuro excerpt #4.

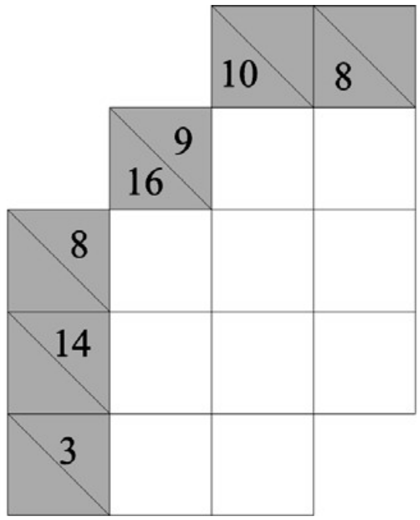
11. Try to solve the full Kakuro puzzle in the figure below.



Junior Kakuro puzzle #2.

12. What did you find difficult about this puzzle? What did you find easy? Did you find any new strategies?

13. Can you solve the excerpt of the puzzle in the figure below? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.



Kakuro excerpt #5.

By now you should be noticing that some clues are more valuable than other simply because the clue number can be broken up in only a few ways. For example, if our clue is a 7 for a run of three then the only possible combination is to use a 1, 2, and a 4. This is quite valuable. Frequent Kakuro players will have many of these examples at hand. For amateurs, like the authors, there are Kakuro Combination Charts available in many places on the Internet.

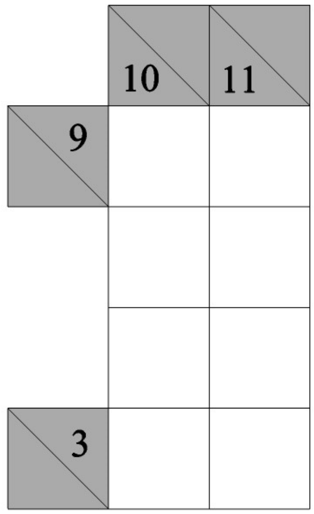
How many different possible combinations a number can take is actually an important mathematical topic. We call 7 = 1+2+4 a *partition* of the number 7. There are 15 different partitions of the number 15. This includes many partitions that are not legal in Kakuro, since digits are repeated, like 7 = 2 + 2 + 1 + 1 + 1. Partitions were first studied systematically by the great **Leonard Euler**. Some of the most important progress was made by the great, but tragically short-lived **Srinivas Ramanujan**. For decades after Ramanujan's death the subject saw mainly minor advances. In 1999-2000, while studying Ramanujan's notebooks, **Ken Ono** made a discovery that shocked the mathematical world - *partition congruences* must exist for every prime number, not just the small family they were thought to hold for. Shortly after his discovery that these partition congruences must exist, one of Ono's undergraduate students, **Rhiannon Weaver**, found 70,000 partition congruences - mathematical patterns that were thought not to exist.

Partitions, counting the number of ways a given whole number can be written as the sum of other whole numbers, are inherently fractal in nature! Their discovery came in a remarkable fashion, as an Emory University press release tells us:

A eureka moment happened in September, when Ono and Zach Kent were hiking to Tallulah Falls in northern Georgia. As they walked through the woods, noticing patterns in clumps of trees, Ono and Kent began thinking about what it would be like to walk amid partition numbers. "We were

standing on some huge rocks, where we could see out over this valley and hear the falls, when we realized partition numbers are fractal," Ono says. "We both just started laughing."[1]
Never underestimate the power of your subconscious.

14. Can you solve the excerpt of the puzzle in the figure below? If so, can this portion of the puzzle be solved uniquely? Whatever your answer, prove that your result is correct.



Kakuro excerpt #6.

15. There are many Kakuro puzzles available online - of all different levels of difficulty. Find one that you think is near the top end of your ability level. Solve this puzzle. Describe the major challenges.

[1] "New theories reveal the nature of numbers," by Carol King available at the http://esciencecommons.blogspot.com/2011/01/new-theories-reveal-nature-of-numbers.html.