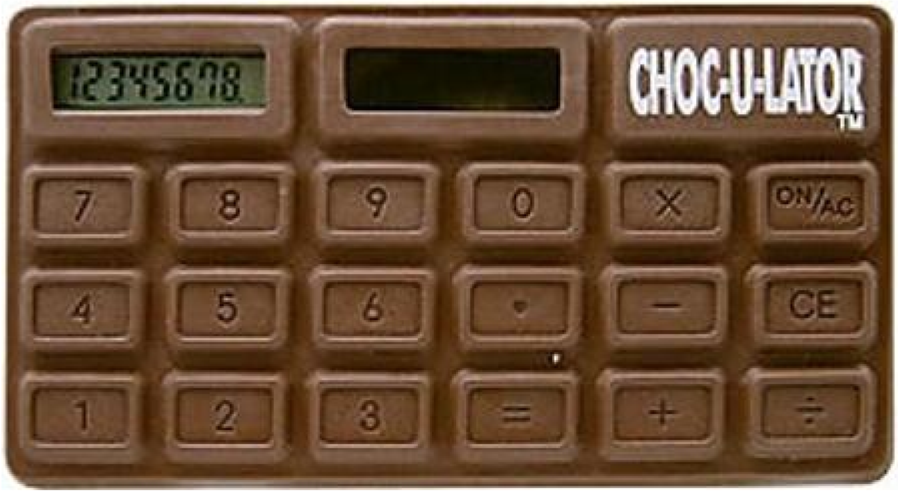

The Chinese Remainder Theorem - Mathematical Magic

A popular online trick is "Calculating Your Age by Chocolate." The trick proceeds via the following steps:

- Choose how many times a week you would like to eat chocolate - any positive whole number.
- Multiply this number by 2.
- Add 5 to the product.
- Multiply the sum by 50.
- Add the current year to the product.
- From this sum subtract 250 if you have already had a birthday this year, otherwise subtract 251.
- From this difference subtract the year of your birth.

1. Pick a number of times a week you want to eat chocolate and perform the steps in the trick.

2. What do you notice about the answer?

3. Repeat the trick with the different starting number.

4. Determine how the trick works algebraically by denoting the starting number by the variable x and performing all of the steps in the trick. Describe how the trick works and any potential limitations on it.

Number tricks based on simple algebraic identities like this have been performed for hundreds of years.

A much more substantial trick based on congruences was described by Fibonacci in his *Liber Abbaci* in 1202. Instead of calling it a "trick" he referred to it as "a pleasant game." But the intent was clear, a method through which "you can know the number said to him in private."

The basis for this trick is the *Chinese remainder theorem* whose earliest known statement appears in the work of **Sun Tzu** (Chinese mathematician; circa 300 - circa 500).

5. Think of any number between 1 and 105. Call your mystery number x.

6. What is x congruent to mod 3? Label your answer as c_1, so we have $x \equiv c_1 \pmod 3$.

7. What is x congruent to mod 5? Label your answer as c_2, so we have $x \equiv c_2 \pmod 5$.

8. What is x congruent to mod 7? Label your answer as c_3, so we have $x \equiv c_3 \pmod 7$.

9. What do the numbers 3, 5, and 7 have to do with 105?

10. Evaluate the expression $m = c_1 \times 35 \times 2 + c_2 \times 21 \times 1 + c_3 \times 15 \times 1$.

11. Reduce m mod 105. Surprised?

12. Do you think that this trick will work for any number between 1 and 105? Explain.

13. Now think of any number between 1 and 231. Call your mystery number x.

14. What is x congruent to mod 3? Label your answer as c_1, so we have $x \equiv c1 \pmod 3$.

15. What is x congruent to mod 7? Label your answer as c_2, so we have$x \equiv c_2 \pmod 7$.

16. What is x congruent to mod 11? Label your answer as $c3$, so we have $x \equiv c_3 \pmod{11}$.

17. Evaluate the expression $m = c_1 \times 77 \times 2 + c_2 \times 33 \times 3 + c_3 \times 21 \times 10$.

18. In the expression for m what do you think gave rise to the numbers 77,33, and 21? Explain.

19. Reduce m mod 231. Surprised?