1.3 More "Sine" Language

The Unit Circle

Clarita is helping Carlos calculate his height at different locations around a Ferris wheel. They have noticed that when they use their formula $h(t) = 30 + 25\sin(\theta)$ their calculator gives them correct answers for the height even when the angle of rotation is greater than 90°. They don't understand why since right triangle trigonometry only defines the sine for acute angles.

Carlos and Clarita are making notes of what they have observed about this new way of defining the sine that seems to be programmed into the calculator.

Carlos: "For some angles the calculator gives me positive values for the sine of the angle, and for some angles it gives me negative values."

1. Without using your calculator, list at least five angles of rotation for which the value of the sine produced by the calculator should be positive.

2. Without using your calculator, list at least five angles of rotation for which the value of the sine produced by the calculator should be negative.

Clarita: "Yeah, and sometimes we can't even draw a triangle at certain positions on the Ferris wheel, but the calculator still gives us values for the sine at those angles of rotation."

3. List possible angles of rotation that Clarita is talking about—positions for which you can't draw a reference triangle. Then, without using your calculator, give the value of the sine that the calculator should provide at those positions.

Carlos: "And, because of the symmetry of the circle, some angles of rotation should have the same values for the sine."

4. Without using your calculator, list at least five pairs of angles that should have the same sine value.

Clarita: "Right! And if we go around the circle more than once, the calculator still gives us values for the sine of the angle of rotation, and multiple angles have the same value of the sine."

5. Without using your calculator, list at least five sets of multiple angles of rotation where the calculator should produce the same value of the sine.

Carlos: "So how big can the angle of rotation be and still have a sine value?"

Clarita: "Or how small?"

6. How would you answer Carlos and Clarita's questions?

Carlos: "And while we are asking questions, I'm wondering how big or how small the value of the sine can be as the angles of rotation get larger and larger?"

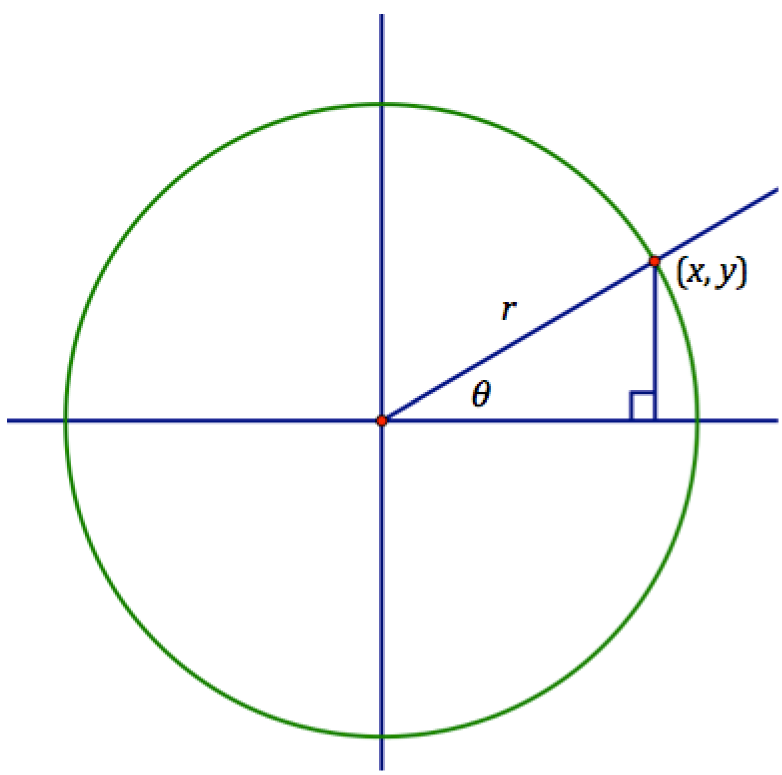
7. Without using a calculator, what would your answer be to Carlos' question?

Clarita: "Well, whatever the calculator is doing, at least it's consistent with our right triangle definition of sine as the ratio of the length of the side opposite to the length of the hypotenuse for angles of rotation between 0 and 90°."

Investigation 2:

Carlos and Clarita decide to ask their math teacher how mathematicians have defined sine for angles of rotation, since the ratio definition no longer holds when the angle isn't part of a right triangle. Here is a summary of that discussion.

We begin with a circle of radius r whose center is located at the origin on a rectangular coordinate grid. We represent **an angle of rotation in standard position** by placing its vertex at the origin, the *initial ray* oriented along the positive x-axis, and its *terminal ray* rotated θ degrees counterclockwise around the origin when θ is positive and clockwise when ⍰ is negative. Let the ordered pair (x, y) represent the point when the terminal ray intersects the circle. (See the diagram below, which Clarita diligently copied into her notebook.)



In this diagram, angle θ is between 0 and 90°; therefore, the terminal ray is in quadrant I. A right triangle has been drawn in quadrant I, similar to the right triangles you have drawn in the Ferris wheel tasks.

8. Based on this diagram and the right triangle definition of the sine ratio, find an expression for *sin* θ in terms of the variables x, y and r.

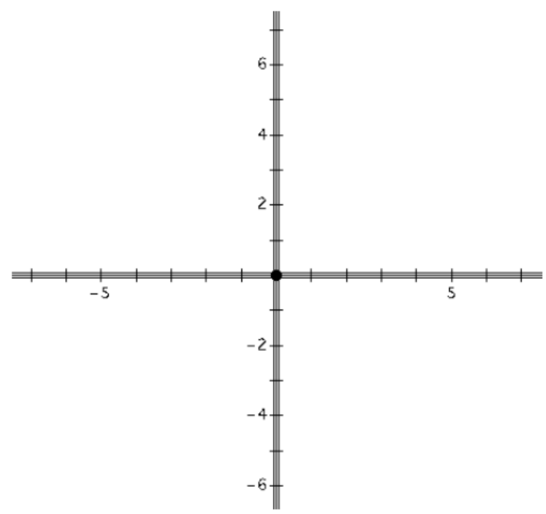
$$\sin\theta =$$

You will use this definition for <u>any</u> angle of rotation. Try it for a specific point on a particular circle.

9. Consider the point (-3, 4), which is on the circle $x^2 + y^2 = 25$.

a) What is the radius of this circle?

b) Draw the circle and the angle of rotation, showing the initial and terminal ray.



c) For the angle of rotation you just drew, what would the value of the sine be if we use the definition we wrote for sine in question 8?

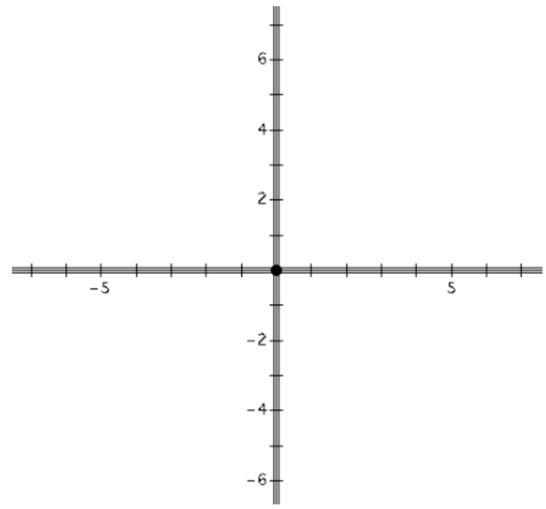
d) What is the measure of the angle of rotation? How did you determine the size of the angle of rotation?

e) Is the calculated value based on this definition the same as the value given by the calculator for this angle of rotation?

10. Consider the point (-1, -3), which is on the circle $x^2 + y^2 = 10$.

a) What is the radius of this circle?

b) Draw the circle and the angle of rotation, showing the initial and terminal ray.



c) For the angle of rotation you just drew, what would the value of the sine be if we use the definition we wrote for sine in question 8?

d) What is the measure of the angle of rotation? How did you determine the size of the angle of rotation?

e) Is the calculated value based on this definition the same as the value given by the calculator for this angle of rotation?