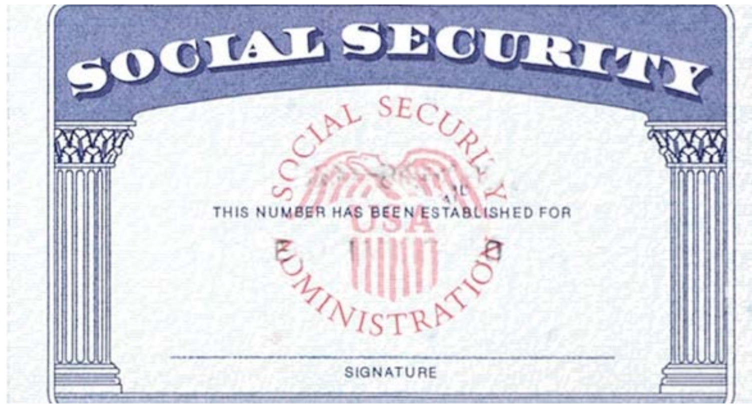
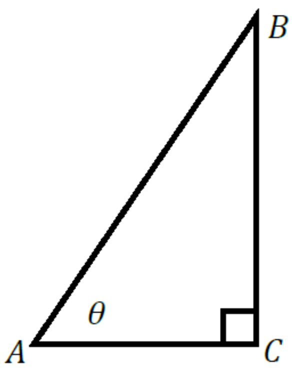
1.14 Maintaining Your Identity

Trig Identities



Right triangles and the unit circle provide images that can be used to derive, explain and justify a variety of trigonometric identities.



For example, how might the right triangle diagram at the left help you justify why the following identity is true for all angles θ between 0° and 90°?

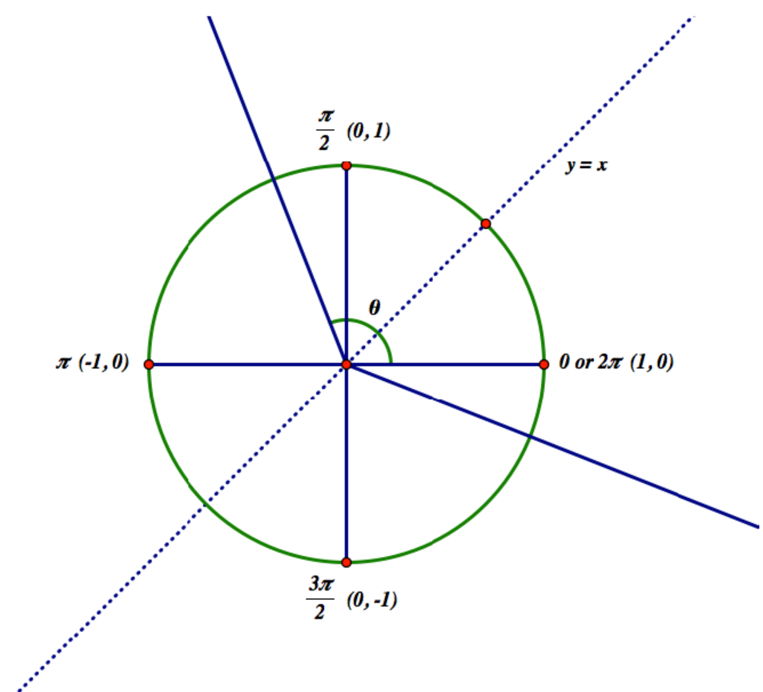
$$\sin\theta = \cos(90° - \theta)$$

Since you have extended your definition of the sine to include angles of rotation, rather than just the acute angles in a right triangle, we might wonder if this identity is true for all angles θ, not just those that measure between 0° and 90°?

A version of this identity that uses radian rather than degree measure would look like this:

$$\sin\theta = \cos(\frac{\pi}{2} - \theta)$$

1. How might you use this unit circle diagram to justify why this identity is true for all angles θ?



Investigation 2: Fundamental Trig Identities

Here are some additional trig identities. Use either a right triangle diagram or a unit circle diagram to justify why each is true.

2. $\sin(-\theta) = -\sin\theta$

3. $\cos(-\theta) = -\cos\theta$

4. $\sin^2\theta + \cos^2\theta = 1$ (Note: this is the preferred notation for $(\sin\theta)^2 + (\cos\theta)^2 = 1$)

5. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

Use right triangles or a unit circle to help you form a conjecture for how to complete the following statements as trig identities. How might you use graphs to gain additional supporting evidence that your conjectures are true?

6. $\sin(\pi - \theta) =$ and $\cos(\pi - \theta) =$

7. $\sin(\pi + \theta) =$ and $\cos(\pi + \theta) =$

8. $\sin(2\pi - \theta) =$ and $\cos(2\pi - \theta) =$

Ycan use algebra, along with some fundamental trig identities, to prove other identities. For example, how can you use algebra and the identities listed above to prove the following identities?

9. $\tan(-\theta) = -\tan\theta$

10. $\tan(\pi + \theta) = \tan\theta$

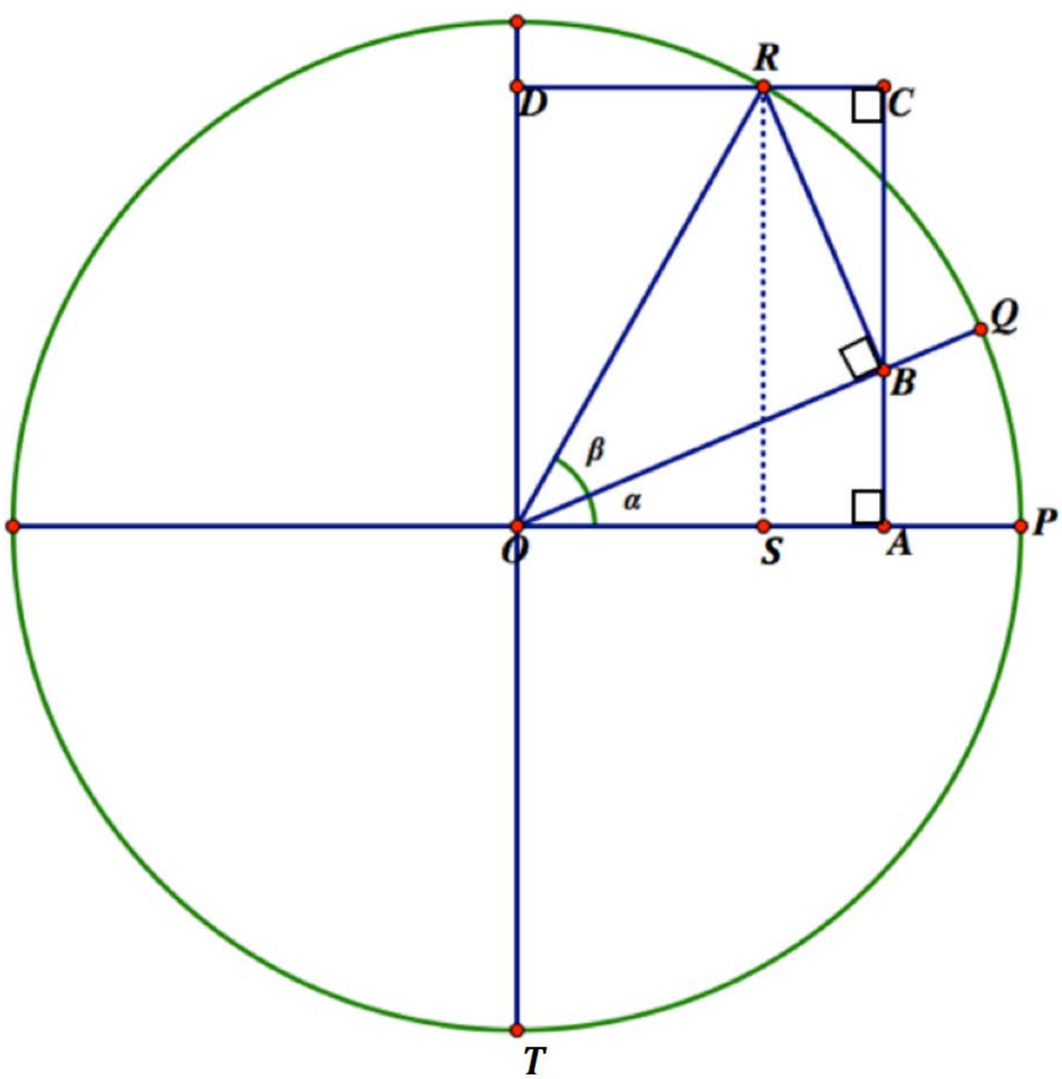
Investigation 3: Sum and Difference Identities

Sometimes it is useful to be able to find the sine and cosine of an angle that is the sum of two consecutive angles of rotation. In the diagram below, point P has been rotated α radians counterclockwise around the unit circle to point Q, and then point Q has been rotated an additional β radians counterclockwise to point R. How are the sine and cosine of angle α, angle β and the sum of the two angles—angle $\alpha + \beta$ —related?

11. Do you think this is a true statement: $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$? Why or why not?

12. Examine the diagram. Figure $OACD$ is a rectangle. Can you use this diagram to state a true relationship that completes this identity? (Hint: label all the segments on the sides of rectangle $OACD$ using right triangle trig relationships.)

$\sin(\alpha + \beta) =$ ______________________



13. Once you have an identity for $\sin(\alpha + \beta) =$ you can find an identity for $\sin(\alpha - \beta) =$ algebraically. Begin by noting that $\sin(\alpha + \beta) = \sin[\alpha + (-\beta\_]$ and apply the identity you found in question 12, along with the identities in questions 2 and 3.

$$\sin(\alpha - \beta) =$$

14. You can also find an identity for $\cos(\alpha + \beta)$ in the diagram above. Since $\overline{OA} \cong \overline{DC}$, and $DC = DR + RC$, using trigonometry to determine the lengths of segments OA, DR and RC will reveal this relationship.

$$\cos(\alpha + \beta) =$$

15. Now you can also complete this identity using reasoning similar to what you did in question 13.

$$\cos(\alpha - \beta) =$$

16. The following identities are known as the double angle identities, but they are just special cases of the sum identities you found above

$$\cos(2\alpha) = \cos(\alpha + \alpha) = \underline{\hspace{3cm}}$$

$$\cos(2\alpha) = \cos(\alpha + \alpha) = \underline{\hspace{3cm}}$$