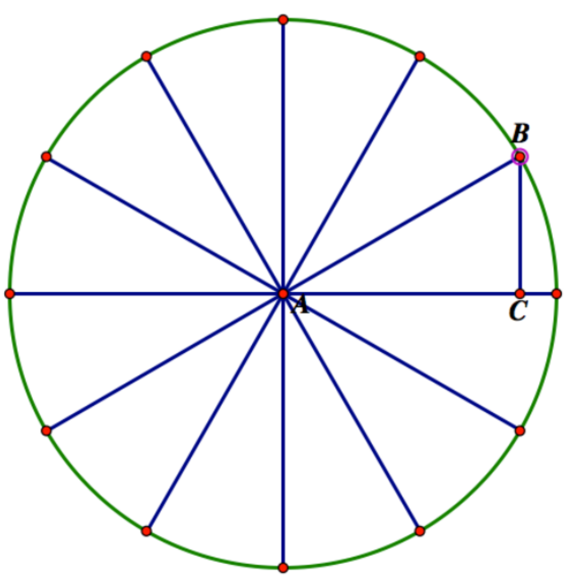
1.9 Water Wheels and the Unit Circle

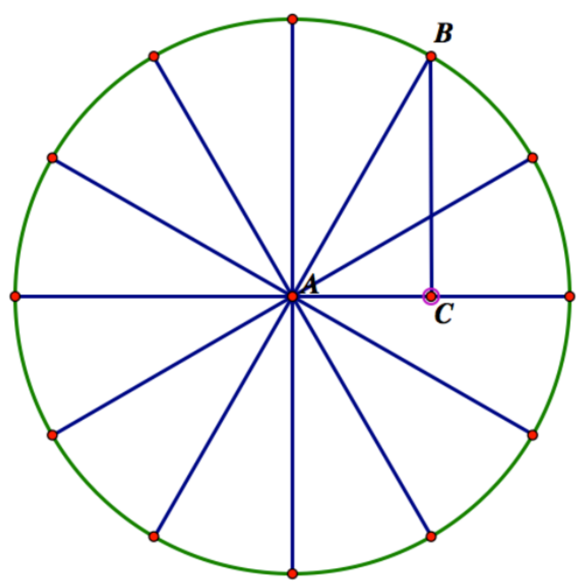
Mastering Radian Measures

Water wheels were used to power flour mills before electricity was available to run the machinery. The water wheel turned as a stream of water pushed against the paddles of the wheel.

Consequently, unlike Ferris wheels that have their centers above the ground, the center of the water wheel would be placed at ground level, so the lower half of the wheel would be immersed in the stream.

The following diagrams show potential designs for a water wheel. Each of the 12 spokes of the water wheel will measure 1 meter. In addition to the spokes, the designer wants to add braces to provide additional strength. Two potential placements for the braces are shown in the following diagrams. (The braces and the spoke to which they are attached form a right angle.)



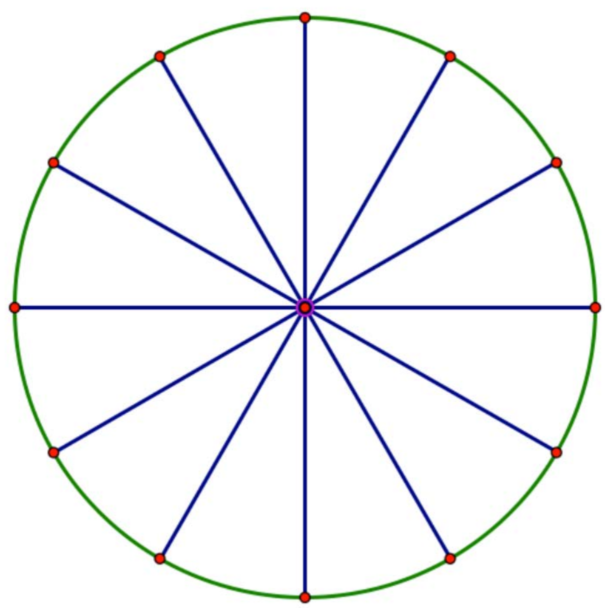


1. Find the measures of $\angle BAC$ and $\angle ABC$ in each diagram.

2. Find the <u>exact lengths</u> of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$, not just decimal approximations. Explain how you found these lengths exactly.

3. Label the exact coordinates of point B in each diagram.

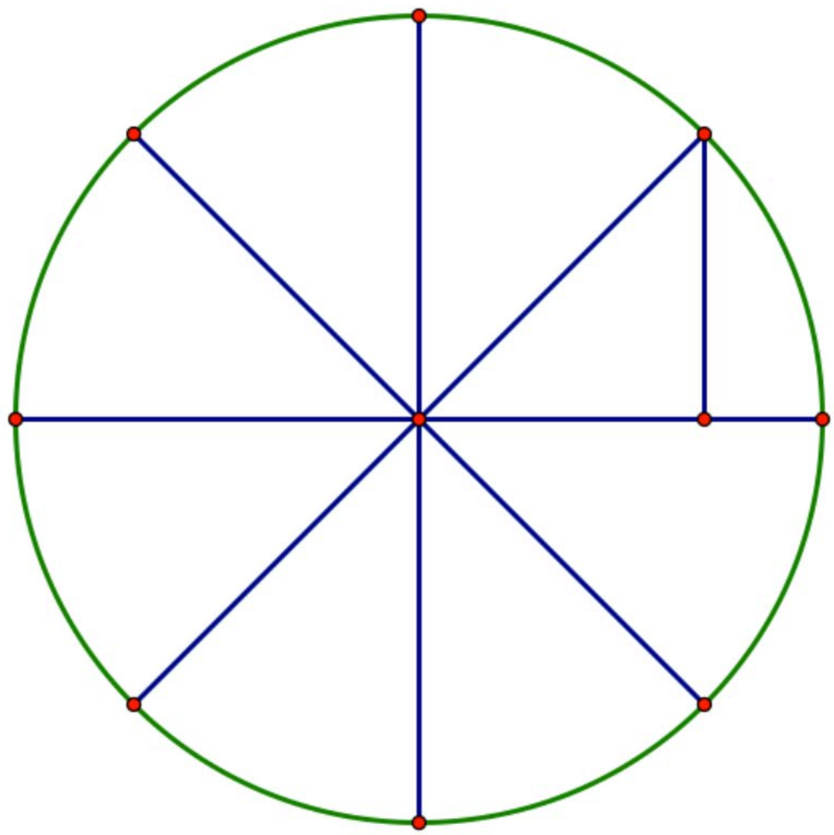
Based on your work above, label the exact values of the x and y-coordinates for each point on the following schematic drawing of the water wheel. Remember that the center of the wheel is at ground level, so points below the center of the wheel should be labeled with negative values. As in the Ferris wheel models, label points to the left of center with negative coordinates also. (Keep for your own reference.)



4. Use the diagram above to give exact values for the following trig expressions.

$\sin\left(\frac{\pi}{6}\right) =$	$\cos\left(\frac{5\pi}{6}\right) =$	$\cos\left(\frac{7\pi}{6}\right) =$
$\sin\left(\frac{\pi}{3}\right) =$	$\cos\left(\frac{\pi}{6}\right) =$	$\cos\left(\frac{11\pi}{6}\right) =$
$\sin\left(\frac{3\pi}{2}\right) =$	$\cos(\pi) =$	$\sin\left(\frac{7\pi}{3}\right) =$

Here is a plan for an alternative water wheel with only 8 spokes. Label the exact values of the x and y-coordinates for each point on the following schematic drawing of the water wheel. (Hint: You might want to begin this work by finding the length of the "brace" shown in the diagram.)



5. Use the diagram above to give exact values for the following expressions.

$\sin\left(\frac{\pi}{4}\right) =$	$\sin\left(\frac{5\pi}{4}\right) =$	$\cos\left(\frac{3\pi}{4}\right) =$
$\cos\left(\frac{\pi}{4}\right) =$	$\cos\left(-\frac{\pi}{4}\right) =$	$\sin\left(\frac{7\pi}{4}\right) =$
$\sin\left(\frac{3\pi}{2}\right) =$	$\cos\left(\frac{3\pi}{2}\right) =$	$\sin\left(\frac{11\pi}{4}\right) =$

6. During the spring runoff of melting snow, the stream of water powering this waterwheel causes it to make one complete revolution counterclockwise every 3 seconds.

a) Write an equation to represent the height of a particular paddle of the waterwheel above or below the water level at any time t after the paddle emerges from the water.

b) Write your equation so the height of the paddle will be graphed correctly on calculator set in *degree* mode.

c) Revise your equation so the height of the paddle will be graphed correctly on calculator set in *radian* mode.

7. During the summer months, the stream of water powering this waterwheel becomes a "lazy river" causing the wheel to make one complete revolution counterclockwise every 12 seconds.

a) Write an equation to represent the height of a particular paddle of the waterwheel above or below the water level at any time t after the paddle emerges from the water.

b) Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

c) Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.