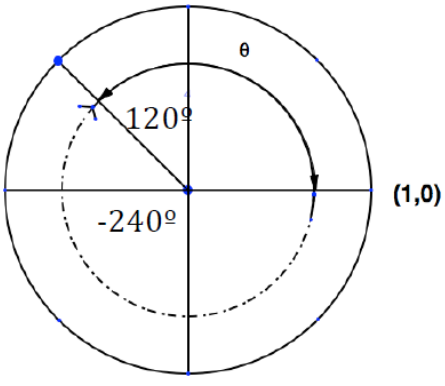
1.9 Water Wheels and the Unit Circle

Practice

I. Coterminal angles.

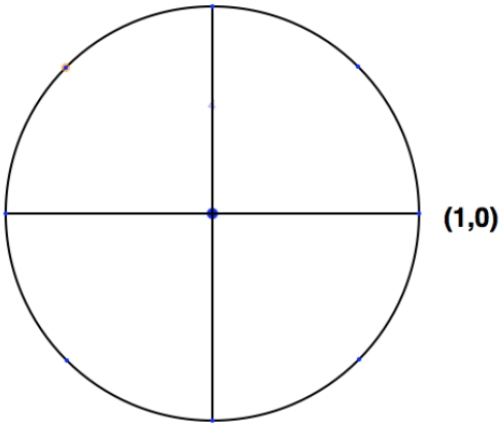
State a negative angle of rotation that is *coterminal* with the given angle of rotation. (*Coterminal* angles share the same terminal side of an angle of rotation.) Sketch and label both angles.



Example 1: $\theta = 120°$ is the given angle of rotation. The angle of rotation is indicated by the solid arc. The dotted angle of rotation is the coterminal angle with a rotation of $-240°$.

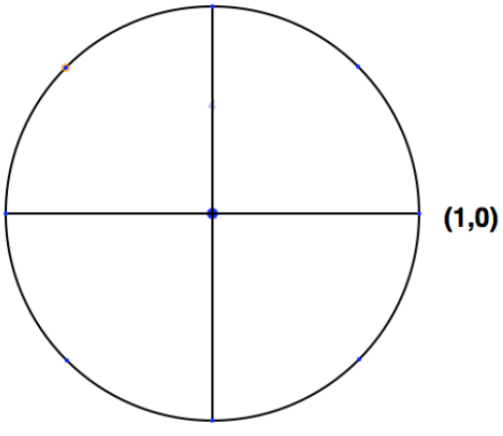
2. Given: $\theta = 95°$

Coterminal angle:



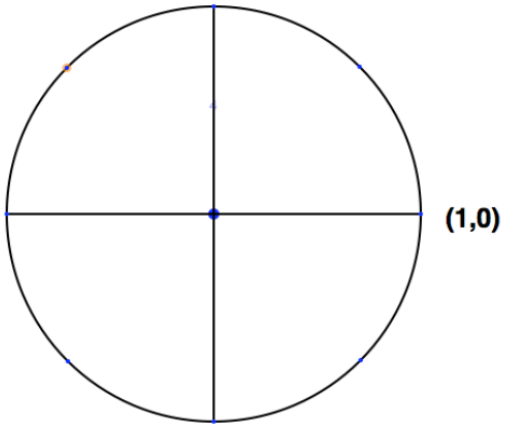
3. Given: $\theta = 225°$

Coterminal angle:



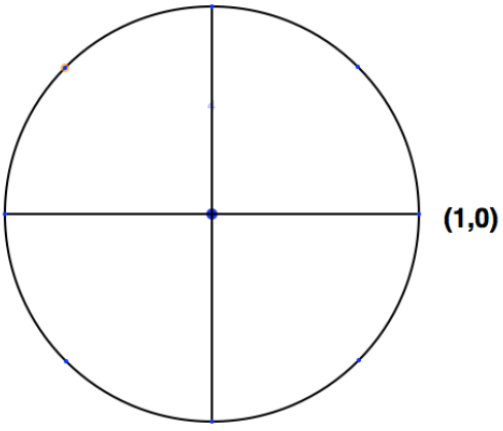
4. Given: $\theta = 95°$

Coterminal angle:



5. Given: $\theta = 95°$

Coterminal angle:



6. What is the sum of a positive angle of rotation and its negative coterminal angle?

7. Every angle has an infinite number of coterminal angles both positive and negative if the definition is extended to angles of rotation greater than 360°. For example: an angle of 45° is coterminal with angles of rotation measuring 405°, 765°, etc.

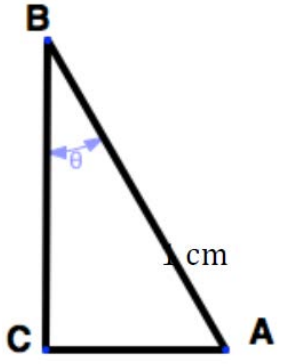
Given $\theta = 115°$, name 3 **positive** coterminal angles.

II. Sine and Cosine of Radian Measures

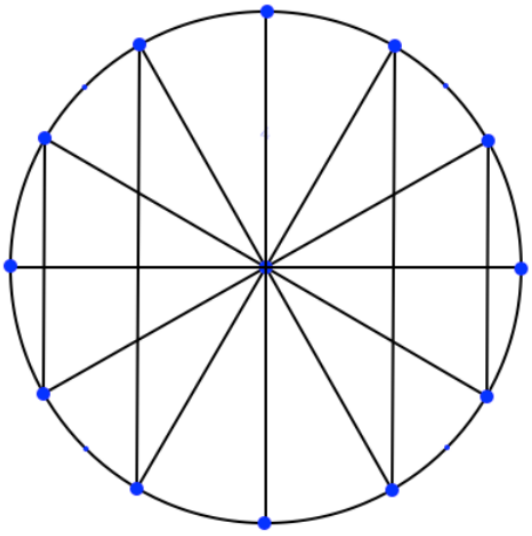
8. Triangle ABC is a 30°, 60°, 90° right triangle.

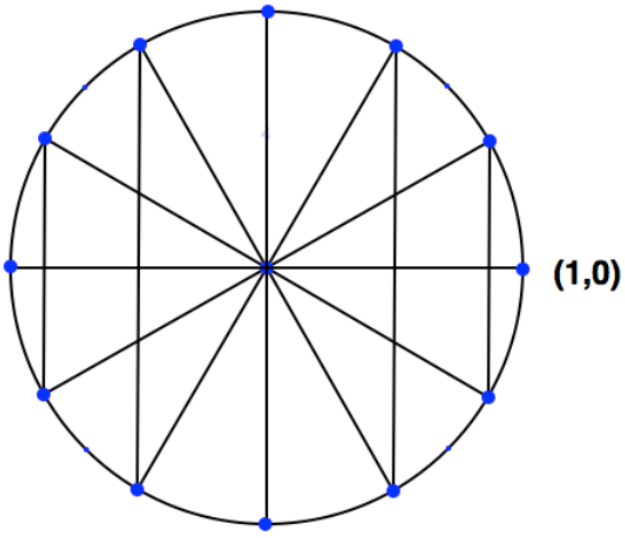
The length of one side is given. Find the values for the missing sides.

$$m\angle B = 30°$$



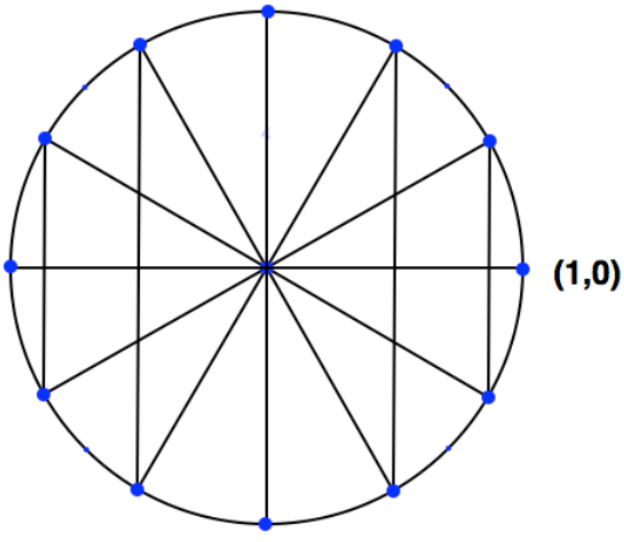
9. Label each point around the circle with the angle of rotation in radians starting from the point (1,0).





10. Use the values in #8 to write the **exact** coordinates of the points on the circle below. Be mindful of the numbers that are negative

11. Find the arc length, *s,* from the point (1,0) to each point around the circle. Record your answers as decimal approximations to the nearest thousandth.



Use your calculator to find the following values

12. $\sin\frac{5\pi}{6}$

13. $\sin\frac{\pi}{3}$

14. Why are both of your answers positive?

15. $\cos\frac{2\pi}{3}$

16. $\cos\frac{4\pi}{3}$

17. Why are both of your answers negative?

18. $\sin\frac{\pi}{2}$

19. $\cos\frac{\pi}{2}$

20. In which quadrants are sine and cosine both negative?

21. Name an angle of rotation where sine is equal to -1

22. Name an angle of rotation where cosine is equal to -1.

III. Inverse Trig Functions

Use your calculator to find the value of θ, where $0 \leq \theta \leq 90°$. Round your answers to the nearest degree.

23. $\sin\theta = 0.82$

24. $\cos\theta = 0.31$

25. $\cos\theta = 0.98$

26. $\sin\theta = 0.39$

27. $\sin\theta = 1$

28. $\cos\theta = 0.71$

When you are looking for the correct angle, as you are in these problems, there are two notations that can indicate the same thing.

- $\sin^{-1}\frac{1}{2}$ This problem is the *inverse sine function*. It is asking for the angle that would make the equation true. Your answer would be 30º in right angle trigonometry. In circular trig it would be 30° *and* 150° because both values make it true. (There are many more correct answers in circular trig if the domain is not restricted. Let's keep it restricted to $0 \leq \theta \leq 90°$.)

- $\arcsin\frac{1}{2}$ This notation is also asking for the angle that would make the equation true.

IV. Assessment – Khan Academy

1. Complete the following online worksheet in the Trigonometry unit of Khan Academy's Precalculus course:
   a. https://www.khanacademy.org/math/precalculus/trig-equations-and-identities-precalc/inverse-trig-functions-precalc/e/inverse_trig_functions