3.3: A Delicate Operation

Vector Operations in Magnitude-Direction Form

The vector addition and subtraction we have done so far has been quite easy, since we could do them **component-wise.** That is, we just added or subtracted each component separately. But if vectors are given in magnitude-direction form, this process becomes somewhat more difficult.

1. Starting from Hogsmeade Station (0,0), Ron's clumsy owl Errol flies 5 miles at a 30° angle (remember, these are standard position angles, so they are measured counterclockwise from East). Then he realizes he's flying the wrong way, so he adjusts his path and flies 3 miles at a 110° angle (again, counterclockwise from East). Hedwig has a better sense of direction, so she flies from Hogwarts directly to Errol's ending point. Give directions to Hedwig to find Errol.

STEP-BY_STEP: In GeoGebra,

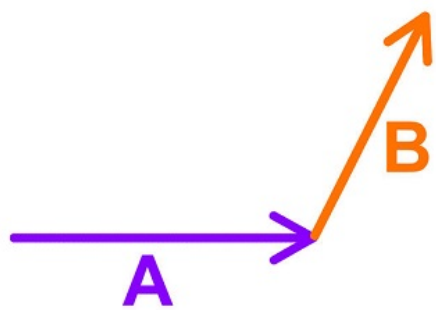
- Use the **Segment Tool** to draw a segment from A(0,0) to B(5,0).
- Use the **Rotate around Point Tool** (Button #9) and select, in order: Point B (the object), Point A (the center) and enter 30° in the dialog box.
    - The rotated point is now labeled B'.
- Use the **Vector Tool** to draw the vector from A to B'. (This is Errol's first vector.)
- Use the **Segment with Given Length Tool**, select B' and enter the new heading: 110°.
    - The endpoint of the new segment is C(7.1,2.87)
- Use the **Rotate around Point Tool** and select, in order: Point C (the object), Point B' (the center) and enter 110° in the dialog box.
    - The rotated point is now labeled C'.
- Use the **Vector Tool** to draw the vector from B' to C'. (This is Errol's second leg.)

- Use the **Vector Tool** to draw the resultant vector from A to C'
    - This is Hedwig's route. We could use the Laws of Sines and Cosines to find the resultant angle and magnitude, but (for now)lets's ask GeoGebra to measure these values instead:
        - Magnitude:
            - Select and delete vector *w*
            - Use the **Segment Tool** to draw $\overline{AC'}$
            - Use the **Distance or Length Tool** and select $\overline{AC'}$.
                - The measurement will be shown as a label

- Angle:
  - Use the **Angle Tool** (also Button 8) and select, in order: B, A and C'.
    - Again, the measurement will be shown as a labeled arc

a. Use your work from above to answer: For Hedwig to fly in a direct path to meet Errol, Hedwig should fly approximately ________ miles at __________°.

This geometric method of adding two or more vectors is known as the **Tip-To-Tail Method**. (It is also known as the Triangle Method, when there are only two vectors being added.)



2. Let's go back to Hedwig's flight. We could only *estimate* Hedwig's path using the tail-to-head method graphically. Let's use trigonometry to get more accurate results. Complete the table below to convert both parts of Errol's flight into component form:

	Magnitude-Direction Form (Owl Form)		Component Form (Harry's Form)	
	Magnitude	**Direction**	**Horizontal**	**Vertical**
Part 1	5 miles	30°		
Part 2	3 miles	110°		

<u>Hints</u>: How far East or West must Hedwig fly to get to Errol's location from Hogwarts? How far North or South?

3. Find the magnitude and direction of Hedwig's path. Compare to your estimates from #2.

4. How far did Errol fly altogether? Did Hedwig fly the same distance? Why does this make sense?

5. There is one particular situation where the sum of the magnitudes of two (or more) vectors *equals* the magnitude of their sum. Describe this situation by drawing a picture and explaining why the sum of the magnitudes equals the magnitude of the sum.

6. Is the direction (angle) of Hedwig's path equal to the sum of the angles of Errol's paths?

7. Errol flies from Hogsmeade Station (0,0), at a 130° for a distance of 5 units. Then he flies at an angle of 65° for a distance of 7 units.

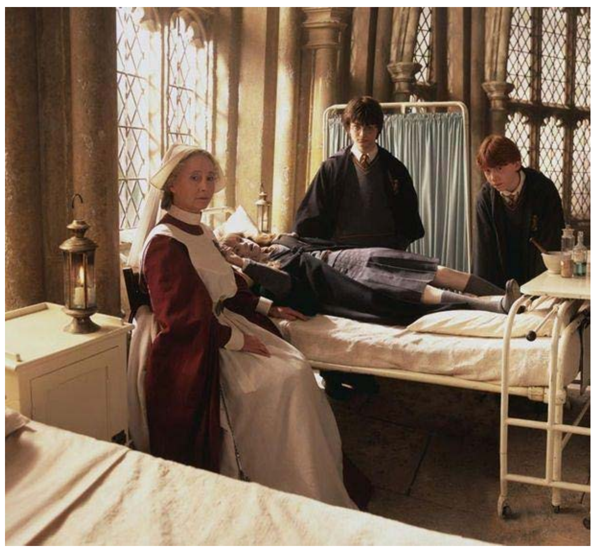
   a. Write the resultant in direction-magnitude form.

   b. Explain why adding the two angles together (130° + 65°) doesn't produce the resultant angle.

8. Errol flies from Hogsmeade Station (0,0), at a 25° for a distance of 2 units. Then he flies 3 units at an angle of 65°. Then he flies 4 units at an angle of 90°. Finally, he flies 3 units at an angle of 165°.

    a. Write the resultant in direction-magnitude form.

    b. Explain why adding the four magnitudes together (2+3+4+3) doesn't produce the resultant magnitude.



II. Another Delicate Operation

Let's see how we can modify the tail-to-end method of addition to apply to vector <u>subtraction</u>.

9. Use component-wise subtraction to find ⟨3, -7⟩ – ⟨4, -5⟩.

10. In GeoGebra, use the Vector Tool to draw the vector $u = \langle 3, -7 \rangle$. (It doesn't matter what initial point you use.) Use the Vector Tool to draw the vector $v = \langle 4, -5 \rangle$ tip-to-tail with vector u. Rotate vector v 180°. Use the Vector Tool to draw the vector from the tail of vector u to the tip of the rotated vector v'.

    a. What is the resultant vector?

    b. Describe why this modified tip-to-tail method will make the resultant vector equal to your answer from #9.

    c. Explain how you know that $v' = -v$.

11. How is subtracting vectors similar to the common "add the opposite" method of subtracting integers?

12. What is the advantage of presenting this subtraction problem in component form rather than direction-magnitude form?

III. An Even More Delicate Operation

Now let's look at scalar multiplication in each form.

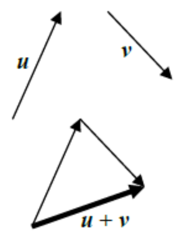
13. From Hogwarts, Harry walks two blocks East and three blocks North then rests. He repeats this five times total. Describe how Ron can walk to Harry's ending position from Hogwarts without stopping.

14. From Hogwarts, Hedwig flies four miles at a direction of 70° then rests. He repeats this five times total. Describe how Errol can fly to Hedwig's ending position from Hogwarts.

15. We have seen that vector addition and subtraction are easier when the problem is presented in component form. Is the same true for scalar multiplication? Explain your answer.

IV. Summary

16. In each box, explain how to perform the indicated operation depending on the form in which the problem is presented. Two boxes have been completed for you.

	Add **u + v**	**Subtract** **u - v**	**Multiply** **au (a is Real)**
Given in component form		*subtract corresponding components*	
Given in Direction-Magnitude Form			
Geometric Representation			
			Stretch if: $\|a\| >$ ___ Shrink if: _________ Reverse direction if: ________________