3.4: 3-Way Tug of War… and More

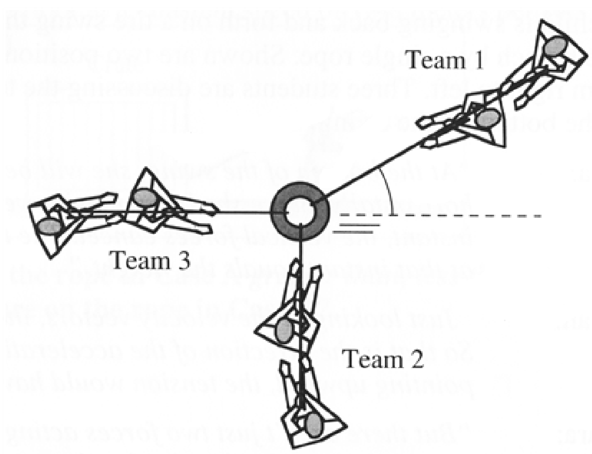
Problem Solving with Vectors

It is easy to determine the winner of a normal tug of war: the team that pulls the other team across the line wins. There are two opposite forces in the rope and the team that pulls the hardest wins.

The study of forces in systems like this is called "Statics" and is an extremely important part of engineering.

1. There are <u>two teams</u> playing a tug-of-war match. At the beginning of the game, they are very evenly matched and are pulling with equal force in opposite directions. Sketch a graph that models this situation and label the vectors ***u*** and ***v***.

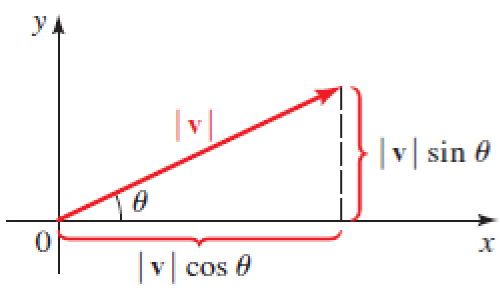
With a 3-way tug, it is not so obvious who the winner will be. You will investigate a 3-way tug of war and calculate the resultant force.



Let's start with a simpler problem:

EXAMPLE 1: Two ropes are pulling on the same tree stump. Vector u represents a force of 200 units pulling in a direction of 100°. Vector v represents a force of 500 units pulling in a direction of 30°. Find the magnitude and angle of the resultant?

- Find the components of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.
  - Vector v
    - Horizontal component $= |v| \cos\theta = 500 \cos 30° \approx 433.013$ units.
    - Vertical component $= |v| \sin\theta = 500 \sin 30° = 250$ units.
    - $v \approx \langle 433.013\,, 250 \rangle$



  - Vector u
    - Horizontal component $= |u| \cos\theta = 200 \cos 100° \approx -34.7296$ units.
    - Vertical component $= |u| \sin\theta = 200 \sin 100° \approx 196.962$ units.
    - $u \approx \langle -34.7296\,, 196.962 \rangle$

- Use vector addition to find the resultant of $\boldsymbol{u} + \boldsymbol{v}$.
  - $u + v \approx \langle -34.7296 + 433.013\,, 196.962 + 250 \rangle \approx \langle 398.283\,,\ 446.962 \rangle$

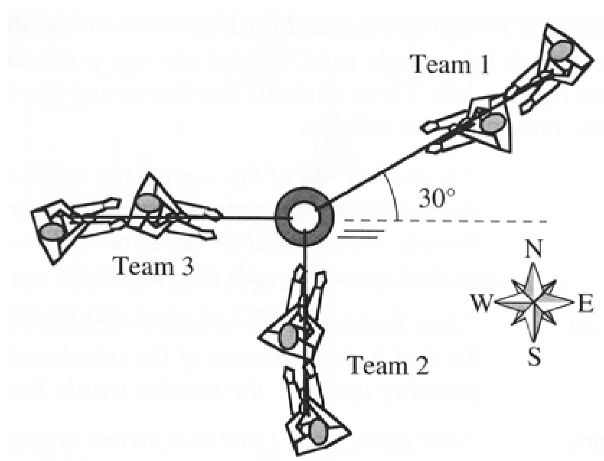
- Find the magnitude of the resultant force
  - $\sqrt{398.283^2 + 446.962^2} \approx 598.669$ units

- Find the direction angle of the resultant force
  - $\tan^{-1}\frac{y}{x} = \tan^{-1}\frac{446.962}{398.283} \approx 48.2961°$

SOLUTION: The stump will be pulled at a resultant force of approximately 599 units of force at an angle of approximately 48.3°

2. Two ropes are pulling on the same tree stump. Vector *u* represents a force of 400 newtons pulling in a direction of 120°. Vector *v* represents a force of 300 newtons pulling in a direction of 40°. Find the magnitude and angle of the resultant?

3. Three teams are playing tug of war as illustrated in the diagram below. Find the magnitude and angle of the resultant force on the tire in the center. The teams are pulling with the following force and direction:
   Team 1 is pulling with a force of 120 newtons at a 30° angle (standard position)
   Team 2 is pulling with a force of 115 newtons at a 270° angle
   Team 3 is pulling with a force of 110 newtons at a 180° angle

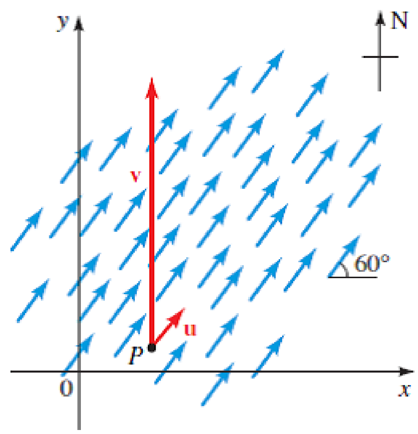


II. Using Vectors to Model Velocity

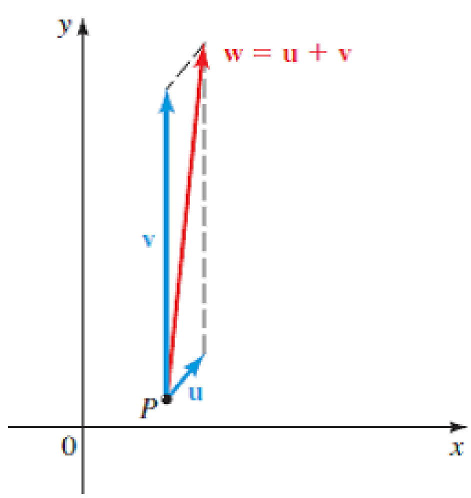
The **velocity** of a moving object is modeled by a vector whose direction is the direction of motion and whose magnitude is the speed. It's obvious that wind affects both the speed and the direction of an airplane.

The graph below shows some vectors **u**, representing the velocity of wind flowing in the direction N 30° E, and a vector **v**, representing the velocity of an airplane flying through this wind at the point *P*.

> Note: Be sure to translate compass directions into standard angle positions – measured counterclockwise from the positive x-axis. (N 30° E is the same as 60°, as marked below.)



The graph below indicates that the true velocity of the plane (relative to the ground) is given by the vector $\mathbf{w} = \mathbf{u} + \mathbf{v}$.



EXAMPLE 2 – The True Speed and Direction of an Airplane

An airplane heads due north at 300 mi/h. It experiences a 40 mi/h crosswind flowing in the direction N 30° E, as shown on the previous page.

a. Express the velocity *v* of the airplane relative to the air, and the velocity *u* of the wind, in component form.

b. Find the true velocity of the airplane as a vector.

c. Find the true speed and direction of the airplane.

SOLUTION

a. The velocity *v* of the airplane relative to the air, and the velocity *u* of the wind, in component form, are

- $v = \langle 0, 300 \rangle$
- $u = \langle 40 \ \cos 60° \ , 40 \sin 60° \rangle = \langle 20 \ , 20\sqrt{3} \rangle$

b. Find the true velocity of the airplane as a vector is given by the vector $\mathbf{w} = \mathbf{u} + \mathbf{v}$.

- $w = u + v = \langle 0 + 20 \ , 300 + 20\sqrt{3} \rangle \approx \langle 20 \ , 334.641 \rangle$

c. The true speed of the airplane is given by the magnitude of **w.**

- $|w| = \sqrt{20^2 + 334.641^2} \approx 335.2$ miles per hour

The direction of the airplane is the direction θ of the vector **w.** The angle θ has the property that

- $\tan^{-1} \frac{y}{x} \approx \tan^{-1} \frac{334.641}{20} \approx 86.6°.$

Answer: the airplane is heading in the direction N 3.4° E at a speed of 335.2 mph.

Your Turn. For each problem below, draw and label a diagram, then use what you know about vectors to answer the question. Show all work on a separate piece of paper.

4. A plane is traveling at 400 mph along a path 40° North of East. A strong wind begins to blow at 50 mph from North to South. If no adjustment is made for the wind, what are the resulting bearing and groundspeed of the plane?

5. You jump into a river intending to swim straight across to the other side. But when you start swimming, you realize the current is traveling 4 miles per hour due south. You are trying to swim due East at 1 mile per hour, but the current is pulling on you. If you don't make any adjustment for the current, how far from your starting point will you be in 15 minutes?

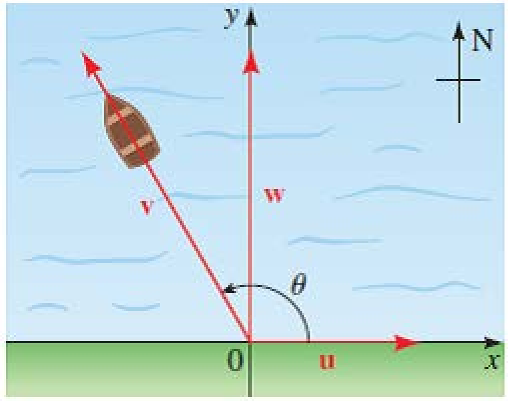
6. A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.
    a. What is the resultant velocity?
    b. If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?
    c. What distance downstream does the boat reach the opposite shore?

III. An Even More Delicate Operation

Rather than having your velocity (and direction) altered by external forces like wind and currents, most navigators adjust their **heading** to end the trip at the expected destination.

EXAMPLE 3: Calculating a Heading

A woman launches a boat from one shore of a straight river and wants to land at the point directly on the opposite shore. If the speed of the boat (relative to the water) is 10 mi/h and the river is flowing east at the rate of 5 mi/h, in what direction should she head the boat in order to arrive at the desired landing point?



S O LU T I O N We choose a coordinate system with the origin at the initial position of the boat as shown in the figure above. Let **u** and **v** represent the velocities of the river and the boat, respectively. Clearly, $u = \langle 5 , 0\rangle$, and since the speed of the boat is 10 mi/h, we have $|v| = 10$, so $v = \langle 10 \cos\theta , 10 \sin\theta\rangle$ where the angle θ is as shown above. The true course of the boat is given by the vector **w** = **u** + **v**. We have

$$w = u + v = \langle 5 + 10\cos\theta , 10 \sin\theta\rangle$$

Since the woman wants to land at a point directly across the river, her direction should have horizontal component 0. In other words, she should choose u in such a way that

$$5 + 10\cos\theta = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\cos^{-1}\frac{1}{2} = 120°$$

Therefore, she should head the boat in the direction 120° or N 30° W.

<u>Your Turn</u>

7. A ship sails 12 hours at a speed of 8 knots (nautical miles per hour) at a heading of 68° south of east. It then turns to a heading of 75° north of east and travels for 5 hours at 15 knots.

    a. Find the resultant displacement vector. Give your answer in component form.

    b. Convert your answer to magnitude-direction form.