3.5: He Who Must Not Be Named

Vectors as Transformations

Voldemort wants to disrupt mail delivery to the wizards at Hogwarts. He has cast a spell that causes each owl to misinterpret directions to its destination.

The spell can be represented as a **transformation matrix**:

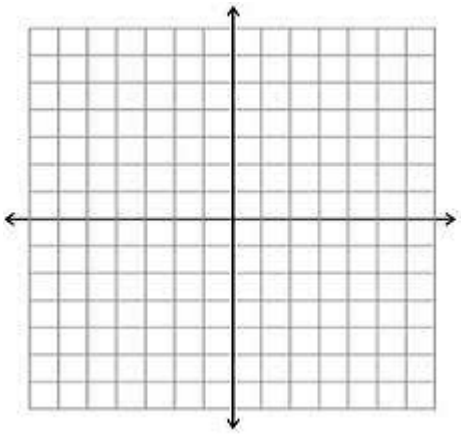
$$T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

To apply this transformation, the owl's directions (in component form) is written as a 2×1 matrix *d*, then the owl follows the directions of the product *Td*.

To get a feel for how this spell works, let's see what Hedwig does when we give her certain directions. If we tell Hedwig to travel 3 miles East and 5 miles South, her direction vector would be written as the 2×1 matrix $d = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

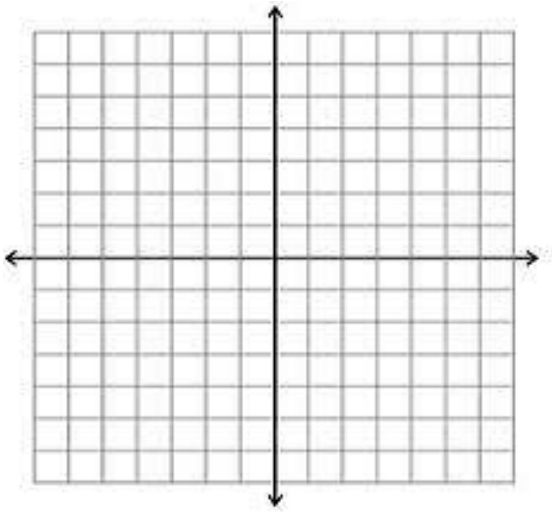
1. The spell will multiply the transformation matrix by this vector. Perform the calculation.

2. On this grid, graph a vector representing the directions we gave Hedwig: ⟨3, -5⟩. In a different color, graph the "spell" vector you found in #1.



3. Repeat for the following. Graph the original and the spell vector on the same grid. Complete the missing values of the table.

Original Directions	d	Magnitude and direction of *d*	Td	Spell vector	Magnitude and direction of *Td*
4 miles West 3 miles North	$\begin{bmatrix} -4 \\ 3 \end{bmatrix}$		$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -4 \\ 3 \end{bmatrix}$		
2 miles West 5 miles North					



4. Did the spell change the length (magnitude) of Hedwig's flight?

   a. Did it change the direction of her flight?

II. Using Vectors to Model Transformations

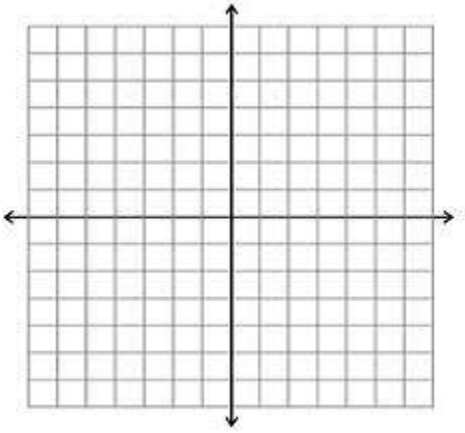
Dumbledore figures out what is going on, so Voldemort has to change his spell. The new transformation matrix is $U = \begin{bmatrix} -1 & -2 \\ 2 & -1 \end{bmatrix}$.

5. Use the transformation matrix to find the "spell vector" when we tell Hedwig to fly 3 miles West and 1 mile South.

6. Did the transformation change the vector's magnitude? Did it change its direction? Justify your answers by showing your calculations for magnitude and direction *and* by graphing the original vector and the spell vector below.

Calculations for magnitude and direction:

Graph:

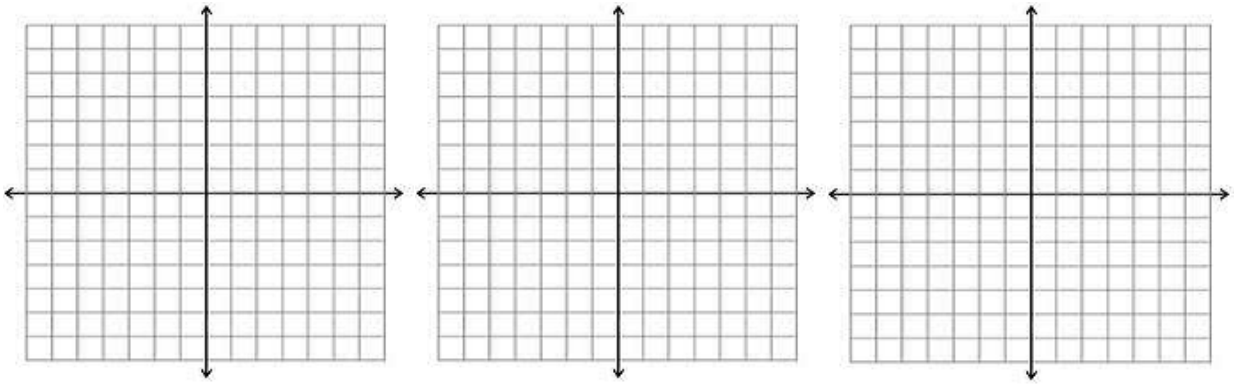


7. Perform transformations on the vector ⟨-3, -1⟩ using each of the following transformation matrices. Graph the original vector and the "spell vector" on the same graph, and state whether the vector's magnitude and/or direction has changed.

Matrix $V = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $W = \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$ $Z = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Spell [ ] [ ] [ ]

Graph



changed… magnitude direction magnitude direction magnitude direction

III. Reversing Spells

Hermione (who was a good student in Arithmancy) knows how to reverse a spell using inverse matrices.

8. Write an inverse matrix that will "undo" the following spells?
   a. $T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
   b. $U = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$
   c. $V = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$
   d. $W = \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$
   e. $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$