8.3 Rational Behavior

Practice Tasks

I. Concepts and Procedures

1. Analyze the end behavior of both functions.

a. $f(x) = x,\ g(x) = \frac{1}{x}$.

b. $f(x) = x^3,\ g(x) = \frac{1}{x^3}$.

c. $f(x) = x^2,\ g(x) = \frac{1}{x^2}$.

d. $f(x) = x^4, g(x) = \frac{1}{x^4}$.

e. $f(x) = x - 1, g(x) = \frac{1}{x-1}$.

f. $f(x) = x + 2, g(x) = \frac{1}{x+2}$.

g. $f(x) = x^2 - 4, g(x) = \frac{1}{x^2-4}$.

2. For the following functions, determine the end behavior.

a. $f(x) = \frac{3x-6}{x+2}$.

b. $f(x) = \frac{5x+1}{x^2-x-6}$.

c. $f(x) = \frac{x^3-8}{x^2-4}$.

d. $f(x) = \frac{x^3-1}{x^4-1}$.

e. $f(x) = \frac{(2x+1)^3}{(x^2-x)^2}$.

f. $f(x) = \frac{5x^6-3x^3+x-2}{5x^4-3x^3+x-2}$.

g. $f(x) = \frac{5x^4-3x^3+x-2}{5x^6-3x^3+x-2}$.

h. $f(x) = \frac{5x^4-3x^3+x-2}{5x^4-3x^3+x-2}$.

i. $f(x) = \frac{\sqrt{2}x^2+x+1}{3x+1}$.

j. $f(x) = \frac{4x^2-3x-7}{2x^3+x-2}$.

3. State the domain of each rational function. Identify all horizontal and vertical asymptotes on the graph of each rational function.
   a. $y = \frac{3}{x^3 - 1}$
   b. $y = \frac{2x+2}{x-1}$
   c. $y = \frac{5x^2 - 7x + 12}{x^3}$
   d. $y = \frac{3x^6 - 2x^3 + 1}{16 - 9x^6}$
   e. $f(x) = \frac{6 - 4x}{x + 5}$
   f. $f(x) = \frac{4}{x^2 - 4}$

4. Sketch the graph of each function in Exercise 1 with asymptotes and excluded values from the domain drawn on the graph.

5. Factor out the highest power of x in each of the following, and cancel common factors if you can. Assume x is nonzero.
   a. $y = \frac{x^3 + 3x - 4}{3x^3 - 4x^2 + 2x - 5}$
   b. $y = \frac{x^3 - x^2 - 6x}{x^3 + 5x^2 + 6x}$
   c. $y = \frac{2x^4 - 3x + 1}{5x^3 - 8x - 1}$
   d. $y = -\frac{9x^5 - 8x^4 + 3x + 72}{7x^5 + 8x^4 + 8x^3 + 9x^2 + 10x}$
   e. $y = \frac{3x}{4x^2 + 1}$

6. Describe the end behavior of each function in Exercise 5.

7. List all of the key features of each rational function and its graph, and then sketch the graph showing the key features.
   a. $y = \frac{x}{x-1}$
   b. $y = \frac{x^2 - 7x + 6}{x^2 - 36}$
   c. $y = \frac{x^3 - 3x^2 - 10x}{x^2 + 8x - 65}$
   d. $y = \frac{3x}{x^2 - 1}$

II. Reasoning

1. Using the equations that you wrote in Exercise 5, make some generalizations about how to quickly determine the end behavior of a rational function.

2. Describe how you may be able to use the end behavior of the graphs of rational functions, along with the excluded values from the domain and the equations of any asymptotes, to graph a rational function without technology.

3. Consider the function $f(x) = \frac{x^3+1}{x}$.
   a. Use the distributive property to rewrite f as the sum of two rational functions g and h.
   b. What is the end behavior of g? What is the end behavior of h?
   c. Graph $y = f(x)$ and $y = x^2$ on the same set of axes. What do you notice?
   d. Summarize what you have discovered in part (b) and (c).

4. Consider the functions $f(x) = x!$ and $g(x) = x^5$ for natural numbers x.
   a. What are the values of $f(x)$ and $g(x)$ for $x = 5,10,15,20,25$?
   b. What is the end behavior of $f(x)$ as $x \to \infty$.
   c. What is the end behavior of $g(x)$ as $x \to \infty$.
   d. Make an argument for the end behavior of $\frac{f(x)}{g(x)}$ as $x \to \infty$.
   e. Make an argument for the end behavior of $\frac{g(x)}{f(x)}$ as $x \to \infty$.