8.7: Dating Woolly Mammoths

Applying Logarithmic and Exponential Functions

Woolly mammoths, an elephant-like mammal, have been extinct for thousands of years. In the last decade, several well-preserved woolly mammoths have been discovered in the permafrost and icy regions of Siberia. Scientists have determined that some of these mammoths died nearly 40,000 years ago using a technique called radiocarbon (Carbon-14) dating.

This technique was introduced in 1949 by the American chemist Willard Libby and is one of the most important tools archaeologists use for dating artifacts that are less than 50,000 years old. Carbon-14 is a radioactive isotope present in all organic matter. Carbon-14 is absorbed in small amounts by all living things. The ratio of the amount of normal carbon (Carbon-12) to the amount Carbon-14 in all living organisms remains nearly constant until the organism dies. Then, the Carbon-14 begins to decay because it is radioactive.

To read more about the carbon dating process, please visit
http://science.howstuffworks.com/environmental/earth/geology/carbon-14.htm

By examining the amount of Carbon-14 that remains in an organism after death, one can determine its age. The half-life of Carbon-14 is 5,730 years, meaning that the amount of Carbon-14 present is reduced by a factor of $\frac{1}{2}$ every 5,730 years.

1. Complete the table.

Years Since Death	0	5,730							
C-14 Atoms Remaining Per 1.0×10^8 C-12 Atoms	10,000								

Let C be the function that represents the number of C-14 atoms remaining per 1.0×10^8 C-12 atoms t years after death.

2. What is $C(11460)$? What does it mean in this situation?

3. Estimate the number of C-14 atoms per 1.0×10^8 C-12 atoms you would expect to remain in an organism that died 10,000 years ago.

4. What is $C^{-1}(625)$? What does it represent in this situation?

5. Suppose the ratio of C-14 to C-12 atoms in a recently discovered woolly mammoth was found to be 0.000001. Estimate how long ago this animal died.

6. Explain why the $C^{-1}(100)$ represents the answer to Exercise 5.

7. What type of function best models the data in the table you created in Exercise 1? Explain your reasoning.

8. Write a formula for C in terms of t. Explain the meaning of any parameters in your formula.

9. Graph the set of points $(t, C(t))$ from the table and the function C to verify that your formula is correct.

10. Graph the set of points $(C(t), t)$ from the table. Draw a smooth curve connecting those points. What type of function would best model this data? Explain your reasoning.

11. Write a formula that will give the years since death as a function of the amount of C-14 remaining per 1.0×10^8 C-12 atoms.

12. Use the formulas you have created to accurately calculate the following:
    a. The amount of C-14 atoms per 1.0×10^8 C-12 atoms remaining in a sample after 10,000 years.

    b. The years since death of a sample that contains 100 C-14 atoms per 1.0×10^8 C-12 atoms.

    c. $C(25{,}000)$

    d. $C^{-1}(1000)$

13. A baby woolly mammoth that was discovered in 2007 died approximately 39,000 years ago. How many C-14 atoms per 1.0×10^8 C-12 atoms would have been present in the tissues of this animal when it was discovered?

14. A recently discovered woolly mammoth sample was found to have a red liquid believed to be blood inside when it was cut out of the ice. Suppose the amount of C-14 in a sample of the creature's blood contained 3,000 atoms of C-14 per 1.0×10^8 atoms of C-12. How old was this woolly mammoth?

II. Other Dating Techniques

Scientists can infer the age of fossils that are older than 50,000 years by using similar dating techniques with other radioactive isotopes. Scientists use radioactive isotopes with half-lives even longer than Carbon-14 to date the surrounding rock in which the fossil is embedded.

A general formula for the amount A of a radioactive isotope that remains after t years is

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where A_0 is the amount of radioactive substance present initially, and h is the half-life of the radioactive substance.

15. Solve this equation for t to find a formula that will infer the age of a fossil by dating the age of the surrounding rocks.

16. Let $A(x) = A_0 \left(\frac{1}{2}\right)^{\frac{x}{h}}$. What is $A^{-1}(x)$?

17. Verify that A and A^{-1} are inverses by showing that $A\left(A^{-1}(x)\right) = x$ and $A^{-1}\left(A(x)\right) = x$.

18. Explain why, when determining the age of organic materials, archaeologists and anthropologists would prefer to use the logarithmic function to relate the amount of a radioactive isotope present in a sample and the time since its death?