# The Binary System of Numeration

**Unit**: The Structure of Number Systems

**Grade**: 9

**Required resources**: Binary number chart - "Let Me Guess Your Age", Overhead projector

**Assumptions**

It is assumed that students will have a general understanding of the concept of different number-base systems. They very well may know how to convert base-10 numbers to other bases. It is not assumed, however, that students will understand the underlying logic and structure of number systems.

**Objectives**

Student will be able to:

1. discover patterns in the binary number system and use those patterns to predict larger numbers
2. apply the concept of grouping to the base-2 number system
3. apply their discoveries to other number bases

**Procedures**

- The teacher will list some number patterns on the board and ask students to solve for the missing numbers. Patterns will reflect addition, subtraction, multiplication and exponential patterns. After the students have time to figure them out at their desks, solutions will be shared and patterns discovered. (10 minutes)
- The teacher will then introduce the question of whether all missing numbers can be discovered through a study of patterns. "Who is the oldest person in this room?", the teacher will ask. "Who is the youngest?" The class may or may not be able to figure these questions out. While interest is high, the teacher will challenge the students to arrange themselves around the room from oldest to youngest person. After the class has completed this task, students will be asked to reflect on their problem solving method - which was not to find a pattern. Could we find out someone's age

without them telling us? Eventually, the teacher would announce that he/she knew a magic trick to do this. (10 minutes)

- The teacher will hand out individual copies and project the *Binary number chart* overhead to the entire class, and challenge the students to discover the secret of the *"Let Me Guess Your Age"* magic trick. Approximately 5 minutes of the class will be spent demonstrating the trick: Students will be asked to tell which columns their age falls into and the teacher will "instantaneously" discover their age.
- For the next 5 minutes, students will be paired off, and attempt to recreate the magic trick with each other.
- Full class discussion will share observations and discover if any students "figured it out". If some have succeeded, the secret will be shared with all. If not, the teacher will describe the process of adding the top row values for all "Yes it's in that column" responses. Students will volunteer to demonstrate that they can guess the teacher's age, and other ages/numbers. Successful understanding will be demonstrated by the students' ability to answer such questions as "What is the age of a person who's number is in Columns A, B and D?" (or A, B, C and E, etc.) and "If someone was 37 years old, what columns would you expect to find their age listed in?" (5 minutes)
- The last 5 minutes of the class would be preparing the students for the homework problem: Why does this chart work? What is it's structure? Open-ended discussion would lead the students to try and see patterns in the chart.
- Homework: Figure out as many patterns you can find in the rows and columns of the chart. Extend the chart so a person, aged 90 could be accommodated.

**Assessment**

Assessment will be ongoing through subsequent lessons. The day's homework assignment requires both 1) an understanding of how the chart works and 2) the discovery of certain patterns. This is the minimal requirements for the next day's lesson.

# Let me tell your age!

| F | E | D | C | B | A |
|---|---|---|---|---|---|
| 32 | 16 | 8 | 4 | 2 | 1 |
| 33 | 17 | 9 | 5 | 3 | 3 |
| 34 | 18 | 10 | 6 | 6 | 5 |
| 35 | 19 | 11 | 7 | 7 | 7 |
| 36 | 20 | 12 | 12 | 12 | 9 |
| 37 | 21 | 13 | 13 | 13 | 11 |
| 38 | 22 | 14 | 14 | 14 | 13 |
| 39 | 23 | 15 | 15 | 15 | 15 |
| 40 | 24 | 24 | 20 | 18 | 17 |
| 41 | 25 | 25 | 21 | 19 | 19 |
| 42 | 26 | 26 | 22 | 22 | 21 |
| 43 | 27 | 27 | 23 | 23 | 23 |
| 44 | 28 | 28 | 28 | 26 | 25 |
| 45 | 29 | 29 | 29 | 27 | 27 |
| 46 | 30 | 30 | 30 | 30 | 29 |
| 47 | 31 | 31 | 31 | 31 | 31 |
| 48 | 48 | 40 | 36 | 34 | 33 |
| 49 | 49 | 41 | 37 | 35 | 35 |
| 50 | 50 | 42 | 38 | 38 | 37 |
| 51 | 51 | 43 | 39 | 39 | 39 |
| 52 | 52 | 44 | 44 | 42 | 41 |
| 53 | 53 | 45 | 45 | 43 | 43 |
| 54 | 54 | 46 | 46 | 46 | 45 |
| 55 | 55 | 47 | 47 | 47 | 47 |
| 56 | 56 | 56 | 52 | 50 | 49 |
| 57 | 57 | 57 | 53 | 51 | 51 |
| 58 | 58 | 58 | 54 | 54 | 53 |
| 59 | 59 | 59 | 55 | 55 | 55 |
| 60 | 60 | 60 | 60 | 58 | 57 |
| 61 | 61 | 61 | 61 | 59 | 59 |
| 62 | 62 | 62 | 62 | 62 | 61 |
| 63 | 63 | 63 | 63 | 63 | 63 |

# The Binary System of Numeration (Day 2)

**Unit**: The Structure of Number Systems

**Grade**: 9

**Required resources**: Binary number chart - "Let Me Guess Your Age", Overhead projector

**Assumptions**

It is assumed that students will have a discovered how to use the chart issued in the previous day. It is also assumed that the students will have made some discoveries in the patterns in the chart.

**Objectives**

Student will be able to:

1. apply the concept of grouping to the base-2 number system
2. apply their discoveries to other number bases

**Procedures**

- The teacher will project the *Binary number chart* overhead to the entire class, and ask for observations about any patterns represented in the chart. There are many, and all will be valued. (10 min._
- The homework topic will be discussed and illustrated on the board or overhead. (5 min)
- The teacher will point out, or return focus to, the pattern in the first horizontal line (64, 32, 16, 8, 4, 2, 1), and ask the students to examine why the chart works, by drawing the following chart on the board:

64 32 16 8 4 2 1

If students do not discover (on their own) that any whole number up to 63 (or 90) in the new chart, the teacher will ask students to check off which numbers can be added to come up with an example number. Check marks will be made in the space below the numbers that are addends, like below (for the number 83)

| | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
|---|---|---|---|---|---|---|---|

Other examples are used until proficiency with the concept is apparent. The teacher asks for suggestions for other notations in the columns in the checkmarks, wanting a sign in each to make sure (yes/No, + -, x o, , etc.)

- The teacher suggests the following to make the "magic trick" even better: If someone in the class could give a silent signal whenever reference to a column was made, it could be even more amazing. The "guinea pig" could be asked to pick a number and share it with the class, but not the magician. When asked if the number was in column F, column, E, etc., the "guinea pig" would be asked to think yes or no. Secretly, someone in the class could signal the "magician". Possible signaling devices are coughing, switching a flashlight, pulling an ear, etc.
- The game can be played with a previously arranged guinea pig (a student from another class, etc.)
- For the final fifteen minutes, the teacher introduces the "signal" for the base two system: O = no, 1 = yes. An example number is demonstrated like below:

| | | 64 | 32 | 16 | 8 | 4 | 2 | 1 | |
|---|---|---|---|---|---|---|---|---|---|
| 83 | = | yes | no | yes | no | no | yes | yes | |
| | OR | 1 | 0 | 1 | 0 | 0 | 1 | 1 | $=1010011_{two}$ |

- Other examples are shown.
- Homework: A problem sheet of base ten - base two conversions

**Assessment**

Assessment will be made through the homework.