Exploring Similarity through Graphic Design

Unifying Theme: The theme for this unit is exploring Similarity through Graphic Design. The unit shall introduce students to a number of career choices which utilize elements of graphic design. This will provide students with relevance to, and motivation for further mathematics pursuits.

Assumptions

It is assumed that students will have a general understanding of fractions. Some previous exposure to ratios and proportions is helpful, but the lesson could be adapted accordingly if not, thereby making it suitable for younger grades. Students should understand the concept of congruency, at a Level 2 comprehension (Informal Deduction), according to the van Hiele theory.

Grade level(s): 8-10

Topics to be covered

Ratio and proportion
Scale Factors
Congruency and Similarity
Fractions, Decimals and Percentage
Measurement
Estimation and Rounding
Fibonacci numbers, Golden ratios, Golden rectangles
Graphic Design

Required resources: Computer Lab; spreadsheet, desktop publishing and Geometer's Sketchpad software programs, TI-82 calculators, Overhead projector. Optional: AutoCAD software.

Also: scissors, rulers, yardsticks, graph paper and scrap paper.

Also: maps, blueprints, diagrams and various other examples of graphic designs.

Important Features of this unit

Practical real-world applications of mathematics

Develops spatial sense

Encourages problem solving

Links to art, science and technology

Use of computers and calculators

Involves visualization, estimation and prediction

Objectives of the unit

At the end of this unit, students will be able to:

1. distinguish between congruent and similar geometric shapes
2. compute the ratio of two numbers
3. use proportions to solve problems
4. apply the properties of proportional measurements to similar geometric shapes
5. determine the scale factor of similar geometric shapes
6. determine the actual measurements of real objects when given a scaled representation and its scale factor
7. Recognize and determine Fibonacci numbers
8. Apply the golden ratio to design aesthetically pleasing constructions
9. create and/or read a scaled representation of a real-world environment
10. design and draw a blueprint for a home or business floor plan on a computer
11. Create a 3-D model from a 2-D blueprint or floor plan

Unit Outline

The entire unit will take about four weeks, approximately eight-ten math classes of 90 minutes each (based on a block scheduling format). The following is the breakdown of the unit in terms of individual lessons:

Day	Topic(s)	Integrated Activities
Lesson 1:	Introduction to Graphic Design Scale Factor	*Design Samples* *Photo Enlargements* *Art: painting grids* *Advertising: Product wrappers*
Lesson 2:	Scale Drawing Ratio/Proportion	*Reading blueprints* *Taking Measurements* *Floor Plans*
Lesson 3:	Properties of Proportionality	*Geometer's Sketchpad* *Scaling objects on computer* *Excel: Exploring* *Proportions of Similar figures*
Lesson 4:	Fibonaccii Numbers in Nature Golden Ratio	*Science*: *Natural Objects* *Excel: Similar ratios*
Lesson 5:	Similarity Similar Triangles Proving Triangles Similar	*Theatrical Set Design*
Lesson 6:	Construction Independent Projects	*Materials lists* *Cut orders* *Building Models*
Lesson 7:	Graphic Design	*Computer: AutoCAD (optional)*
Lesson 8:	Assessment	

Lesson 2
Learning ratios and proportions through scale drawings.

Required resources:

1/4" and 1/2" graph paper

rulers

yardsticks

scissors

a variety of blueprints, designs and maps

Objectives

Student will be able to:

1. develop a conceptual understand of a ratio as a comparison between two numbers
2. understand that a proportion is an equivalent relation between two ratios
3. understand that scale drawings are ratios
4. measure the dimensions of a room
5. measure various objects in the room
6. make a scale drawing of the classroom
7. place scale model furniture in a room

Procedures

- Introduction (10 minutes): The word 'scale' will be written on the chalkboard and discussed for meaning. Students will come up with as many ideas as possible (weight, temperature, and maps) and the teacher will write them on the board. A map will be examined with its scale identified as ratios.

  Introductory questions:

  How does the ratio affect the size of the map?

  What would happen if the ratio were different?

  Other maps with different scales will be examined.

- Reading Blueprints (20 minutes) Students will work in groups. Each group will be given an identical copy of a blueprint and a ruler and be asked the following questions:
    1. What is the scale factor of the blueprint?
    2. What will the dimensions of each room be, if the house is built as designed?
    3. What is the overall length and width of the house?
    4. What would the overall dimensions be, if the scale factor was 3/16? *(note: rather than 1/16)*
    5. Can you predict what the dimensions of the other rooms would be, given a 3/16 scale factor?
    6. Develop a general formula for converting blueprint measurements(c) to actual dimensions (d) for any scale factor (a/b)

- Full class discussion (15 minutes) will be held, comparing the resulting measurements from each group. Each group's formula shall be written on the chalkboard and explained. The teacher will ask students to try each algorithm for the following situations: (scale factor = 2/9; blueprint measurement of 6), to see if they all agree. The class shall choose the 'best' one. If no group has developed an algorithm similar to the "Cross Product Property of Proportions", it shall be developed on the chalkboard, using measurements from the blueprint. (i.e. If a/b = c/d, then ad=bc)

- Measuring the Classroom (10 minutes) Students will work in pairs. Each pair will use a yardstick and record the measurements for the length and width of the room, table, teacher's desk, chairs and cabinets. Measurements will be compared for accuracy/consistency.

- Floor Plans (30 minutes): The teacher will hand out individual copies and project the *Sample Furniture* overhead to the entire class, and challenge the students to discover the appropriate scale factor which would be required to use the paper furniture as representations of the existing furniture in the classroom. Every student will then draw a floor plan of the classroom on whichever graph paper they choose, which corresponds to the scale factor which was previously determined. When the floor plan is complete, students will be asked to represent the current furniture arrangement of the classroom using the paper models.

- For the final 5-10 minutes (if time is available), students will work in their pairs to "re-design" the classroom for a different furniture arrangement. Pairs can be selected to present their 'designs' to the rest of the class and point out the advantages or disadvantages of such an arrangement.

- Homework: Each student will be asked to re-design the existing classroom as a veterinarian's office given the following equipment/furniture list and measurements:

Examination Table	30" x 66"
Counter	2' x 8'
Scale	1' x 1'
Chair	18" x 18"
Cage	24" x 32"
Waste basket	12" x 18"
Medical Waste Container	15" x 24"
Cabinet	24" x 36"
Desk	27" x 48"

Mathematical Value of the lesson

This lesson involves many important computational skills, such as multiplication and division of fractions. It also requires students to use variables and solve for a missing variable. More importantly, this lesson introduces students to both enactive and iconic representations of ratio and proportion. The entire lesson is developed on the basis of problem solving: the students are presented with inquiries and objectives (determining a scale factor, taking/recording measurements, developing a formula, etc.). They are not given prescribed directions, and the goal is that the students develop their own phenomenological basis for the cross product and reciprocal properties of proportions. Finally, the lesson requires and develops students' skills in visualization and spatial awareness.

Connection of this lesson with other subject areas

This lesson continues with the unifying theme of the unit - i.e. using graphic designs, in the form of maps and blueprints, to explore ratios and proportions. As a result, students will have the opportunity to explore career choices (and the math skills they require) in engineering, architecture, cartography, construction and interior design.

The lesson also continues the connection with the visual arts established in the first lesson, as students are expected to create graphic representations and manipulate models by hand. Towards the end of the lesson, students are asked to utilize their own creativity, as they are asked to develop their own ideas for a better classroom design. This aesthetic element of graphic design is further continued in the homework assignment.

Finally, the lesson can also be connected to social studies (i.e. currency exchanges), Music (Harmonic Means) and Science (optics; converting measurements and forces).

Connection of the lesson with other lessons in the unit

This lesson is designed to follow directly from the student's introduction to Graphic Design in the first lesson. In that lesson, students were able to examine numerous 'designs' representing the work of architects, graphic artists, interior designers, fashion designers, etc. The 'introduction' provides a connective element from the first lesson (the maps), and is designed to provide a transition to the concept of size, scale and proportionality. The first lesson also established a hands-on connection to the art and drawing skills/concepts which are central to the unit. On day one, students were introduced to the concept of rendering a 3-dimensional subject on a 2-dimensional surface. They were also given the opportunity to "draw" a larger version of an existing rendering by using the proportional grid method.

In the third lesson, students will continue to further explore the properties of proportions established in this lesson. First, students will share with the class their veterinarian office designs on an opaque projector. The teacher will then introduce the mathematical terminology and move to a purely symbolic exploration of the examples. The measurements from the homework exercise will be used to demonstrate the Cross Product Property of Proportions on the symbolic level. Students will explore other properties (Reciprocal Property and Interchange Means) through symbolic representation, as well. These fundamental math skills will provide students with the ability to make discoveries about Fibonacci numbers and the golden ratio in Lesson 4 and Proving Similar Triangles in Lesson 5.

This lesson also provides a very important foundation for the student's conceptual exploration of similarity in Lesson 5. Students will be exposed to various computer software programs (both desktop publishing and Geometer's Sketchpad, which allow for manipulation of geometric figures. Students will have the opportunity to create similar shapes, take measurements of corresponding parts and set up ratios to describe the relationship of the corresponding parts. The ratios will be computed through the use of a spreadsheet program (like Excel) and the students will examine patterns to develop their own theory and definition of similarity.

The manipulation of geometric shapes (in the form of paper furniture) also connects with the manipulations later created with the aid of computers; and sets the stage for the students' final projects. The projects are to be original creations for the design of a product, building or other artistic endeavor.

The students' creativity is nurtured and developed throughout the unit by carefully chosen activities which develop from the purely representational renderings of the first lesson through the vets office assignment, and into work with a stronger creative element.

Finally, the unit continually focuses on the real-world applications of ratios and proportions to numerous careers. Architecture shall remain the primary focus, and students will examine the use of proportions in that career from the design process through cost estimation and even in the construction phase. The mathematical concepts (and computational skills with fractions) shall be continuously supported throughout. In addition to the field of architecture, students will gain practical exposure to possible careers in Art (both Graphic and Fine), Photography, Theatrical Set Design, Costume Design, Interior Design, Landscape Design, Fashion Design, and Printing.

Extension of the unit

The real-world career connections allow for numerous opportunities to extend this unit. Field trips and guest artists may be easily integrated into the unit. If time is available, students may be asked to conduct an observation (or an internship) with a particular individual or company which uses some elements of design. These 'field experiences' could be continued throughout the year, with students being asked to periodically, report on connections of their assignment to the current lesson.
These connections would be easy to make, as this unit can easily be related to:

- Planar Measurements
- Space Measurements
- Transformations
- Right Triangles
- Circles
- Tessellation
- Topology
- Statistics and Probability

Assessment

Assessment will be ongoing throughout the unit. The tasks and assignments both build on, and require, a conceptual understanding of mathematical terminology, concepts and computational skills. In addition, students will use journal writing to monitor and clarify their explorations. The final project will also serve to reflect an assessment of the student's progress. Projects will be presented to the entire class, and students will be required to identify (with examples) the mathematics which they used to complete their projects. A more traditional unit test is another, less desirable, option.