

Balloon Toss

Abby and Bing Woo's bakery shop has been so successful (due to the financial advice they received) that they want to return the favor. They decide to hold a carnival to express appreciation for the support of all their customers. One of the carnival booths they want to operate is a water balloon toss, where contestants pay for the opportunity to toss water balloons at local dignitaries.

Mr. Levy, a local math teacher, agrees to be one of the victims - on one condition: He doesn't mind getting wet, but is afraid that some of his Algebra students would take the opportunity to inflict bodily harm. In order to keep the balloon toss safe, Bing Woo suggests that the contestants should be made to toss the water balloons through a raised hoop. This would keep any of the contestants from throwing the water balloon overhand at Mr. Levy.

Your task is to design the booth for the water balloon toss. Your design should include both a drawing of what the booth will look like from the front, and an overhead diagram showing all of the following measurements:

- the dimensions of the booth (there is a total of 30 feet of fencing available)
- the distance between the contestants and Mr. Levy's chair
- the height of the hoop
- the distance of the hoop from the contestants **and** Mr. Levy

Finding the N^{th} Term

The pattern of numbers 3, 4, 5, 6, 7, 8, 9, . . . is an example of a sequence of numbers. The first term of the sequence has a value of three, the second tern has a value of four, the third term has a value of five, and so on. Let's show this as a table.

Term	1	2	3	4	5	6	20	 200	 n
Value	3	4	5	6	7	8	22	 202	 n + 2

Each number in the top row represents a term or position number in the sequence. The number beneath each term is the value of that term of the sequence. The n and n + 2 tell us the formula that connects the top row with the bottom row. This particular formula (n + 20) states that to find the value of any particular term in the sequence, you simply take the number value of the term (n) and add two to that number. Therefore, to determine the value of the 20^{th} term of this sequence, you add 20 + 2 to get 22.

Let's look at another example to see how to find the *n*th term formula:

Example A

Term	1	2	3	4	5	6	20	 n
Value	3	6	9	12	15	18		

To find the formula that describes the sequence above, notice that the value of each term is three times the number of the term (n). According to this pattern, what would be the value of the 20^{th} term? How would you write the formula in terms of n? (hint: what do you do to the term (n) to get its value? How do we write three times some number?)

Now, it's your turn: Find the *n*th term in each sequence below:

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1.								
Term	1	2	3	4	5	6	• • •	n
Value	5	6	7	8	9	10		
2.								
Term	1	2	3	4	5	6		n
Value	- 2	- 1	0	1	2	3		
3.								
Term	1	2	3	4	5	6		n
Value	4	8	12	16	20	24		
4.		_						
Term	1	2	3	4	5	6		n
Value	4	7	10	13	16	19		
5.								
Term	1	2	3	4	5	6		n
Value	3	8	15	24	35	48		

POW 1

Galileo

In class today, you prepared a graph of t (time) and s (the distance an object falls during that time). In this POW, you are going to continue to reproduce Galileo's work and guess the relationship between s and t.

Your task is to write an algebraic expression which describes s (distance) in terms of t.

Your formula needs to turn 1 into 16, 2 into 64, 3 into 144, and so on. In other words, when you substitute (or 'plug in') the number 1 in place of t in your formula, s would have a value of 16; when you substitute 2 for t, s = 64, etc.

In function notation, you want to write:

$$s = f(t)$$

That is, s (distance) is some "function" of t (time); and your task is to describe that functional relationship in an algebraic formula.



Write-Up

You should write up your work on this problem using the following categories:

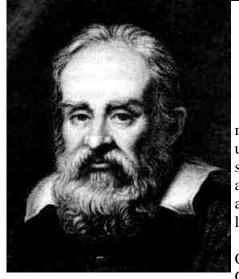
1. Process: Based on your notes, describe what you did in attempting to solve this problem. You might want to talk about such things as approaches you tried that didn't work, what you did when you got stuck, who you talked to and any ideas that you got from them, and so on.

2. Results:

- a. State your results in the form of an algebraic equation using the variables *s* and *t*. Also, be sure to include a description of which values of *t* make sense in this physical problem (of dropping an object from a certain height).
- b. Explain how you know that your answer is correct. Your explanation should be written in a way that will be convincing to someone else.

3. Making Predictions:

- a. Use your formula to predict the distance travelled by a dropped object which falls for 5 seconds, 6 seconds, 10 seconds and 1/2 second.
- b. Describe a real world experiment which could test your prediction for a 10-second drop. Be specific: what object would you use, where would you drop it from?



Galileo Galilei

Galileo Galilei (1564 - 1642) was an Italian astronomer, mathematician and physicist. Galileo was the first scientist to use a telescope to study objects moving in space. He also showed that an object shot into the air follows a **parabolic** path, a special kind of curve. This had practical value at the time, but also led to the discoveries that ultimately resulted in our ability to launch satellites and to explore space.

In Galileo's day, some dismissed his science as magic. Others simply did not want to admit that he was right. Today Galileo is credited with starting the age of modern science. He

formulated problems clearly and did experiments to test his ideas. He believed in simple explanations and in using proven theorems to make predictions. Most importantly, he directed attention to mathematics as the language of physical science.

At 25, Galileo, challenged Aristotle's theory that a heavy object falls faster than a lighter

one. According to legend, he dropped two balls - one ten times heavier than the other - from the top of the Tower of Pisa. Translating his data into our own units of measure, we have the following table of values for his two variables: t, which gives the time, measured in seconds, that the object has been falling, and s, which gives the total distance the object falls, measured in feet. As you can see the s measurements for both objects were identical, proving Aristotle was wrong. However, rather than attempt to explain this phenomenom, Galileo sought a mathematical formula to describe the relationship between the two variables.

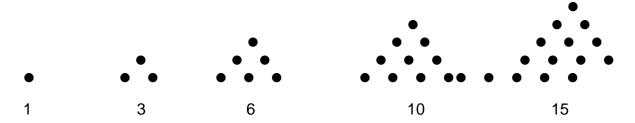
t (in sec.)	s (heavy)	s (light)		
0	0	0		
1	16	16		
2	64	64		
3	144	144		
4	256	256		

Before conducting his experiments, Galileo postulated that the distance fallen was proportional to the time elapsed - in other words: objects fall at the same consistent rate, or speed. If this is true, a linear equation such as s = k t, where k is some fixed constant (equal to the object's speed), would describe and predict the movement of falling bodies. What do you think? Did Galileo's data confirm his hypothesis? Test your answer using the data presented in the table above.

Use a piece of graph paper to plot the points corresponding to the values in the table. Since it seems natural to think of *s*, the distance fallen, as being dependent on *t*, the time the object has been falling, let *s* be the dependent variable measured on the vertical axis, and let *t* be the independent variable measured on the horizontal axis.

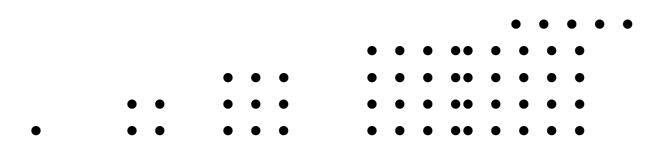
Be sure to label both axes carefully, including: the quantity being measured; and the units in which that quantity is measured (days, square meters, thousands of persons, and so on). You also need to show the scale on each axis. The numbers in the scale should be equally spaced: that is the length of the *s*-axis from 0 to 10, for instance, should be the same as the length of the *s*-axis from 40 to 50.

Picture Patterns



The sequence above appears in many problems. The sequence is called the triangular numbers. The ancient Greeks were probably the first to work with triangular number sequence. Can you predict the values of the next 5 terms in the sequence? Complete the table below. Can you find a formula for the nth term? (Don't despair if you can't - it's tricky!)

Term	1	2	3	4	5	6	7	8	9	10	 n
Value	1	3	6	10	15						



The sequence of numbers above are called square numbers. Can you predict the values of the next 5 terms in the sequence? Complete the table below. Can you find a formula for the nth term? (This one is easier!) Why is this sequence called the square numbers? (Can you describe **both** reasons?)

Term	1	2	3	4	5	6	7	8	9	10	 n
Value	1	4	9								

Find the *n*th term in each sequence below (hint: compare to the square number sequence, above):

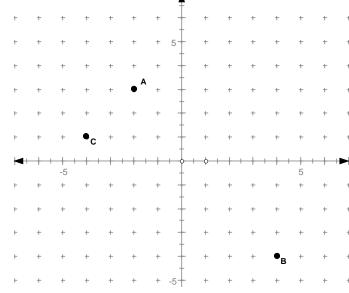
1.							
Term	1	2	3	4	5	6	 n
Value	2	5	10	17	26	37	

2.							
Term	1	2	3	4	5	6	 n
Value	2	8	18	32	50	72	

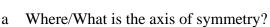
3.							
Term	1	2	3	4	5	6	 n
Value	1	7	17	31	49	71	

Reflections on Quadratics

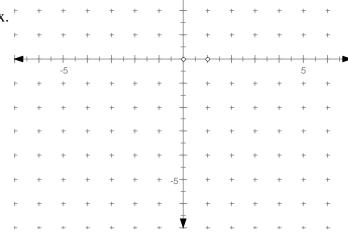
- 1. Use the graph on the right to help you answer the following questions:
- a Name the coordinates of point A. What are the coordinates of its reflection across the y-axis?
- b Name the coordinates of point B. What are the coordinates of its reflection across the y-axis?
- c Name the coordinates of point C. What are the coordinates of its reflection across the y-axis?

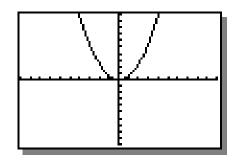


- 2. Write the word or phrase that correctly completes each statement below:
- a The graph of each quadratic function is called a _____
- b The point of intersection of the parabola and its axis of symmetry is the _____
- c Every point on one side of a symmetric figure has a mirror image on the other side of the
- d A mirror image of a point is called its
- 3. Graph $y = 0.5x^2$ on the graph on the right



- b Find and label the coordinates of the vertex.
- c Find and label the coordinates of three other points on the graph **and** the coordinates of the reflections of those points





Exploring Parabolas

Work individually You will need a graphing calculator

Every parabola is the graph of a quadratic function

$$y = 2x^2$$

$$y = x^2 + 2$$

$$y = 2x^2$$
 $y = x^2 + 2$ $y = -3x^2 + 2x - 4$

When a quadratic equation is written in the form
$$y = ax^2 + bx + c$$
 of , it is in **standard form**.

1. Name the values of a, b, and c in each of the quadratic equations below:

a.
$$y = 3x^2 - 2x + 5$$
 $y = 3x^2$ b.

$$y = 3x^2$$
 b.

$$y = -0.5x^2 + 2x^{-C}.$$

2. Write each quadratic function below in standard form, and name its values of a, b and c:

a.
$$y = 7x + 9x^2 - 4$$
 b. $y = 3 - x^2$ c. $2 + y = 3x - 4x^2$

b.
$$y = 3 - x^2$$

c.
$$2 + y = 3x - 4x^2$$

Use your graphing calculator to to see how different values of a, b and c affect the appearance of the graph of a quadratic function.

3. How does changing a affect the graph? (Hint: graph the equations on the right. You may wish to overlay all three graphs at the same time. Add additional graphs as needed to answer the questions below)

$$y = x^2$$

$$y = 2x^2$$

- c) What happens to the graph if you change the sign of a to a negative?
- d) Is there any point on the graph that doesn't change as you change the size of a?
- e) What happens to the graph if a is 0? Is this still a parabola? Is is still a quadratic function?

4. In a similar fashion, investigate how the coefficient b affects the graph of a quadratic function. Write a few sentences about what you discover.

5. In a similar fashion, investigate how the constant c affects the graph of a quadratic function. Write a few sentences about what you discover.

how you decided.

Part A: Without using a graphing calculator, match each function with the graph it describes. Explain

1.
$$y = 3x^2 - 2$$

2.
$$y = -x^2 + 4$$

3.
$$y = x^2 + 4$$

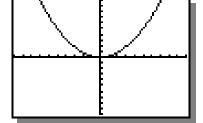
4.
$$y = 0.2x^2$$

5.
$$y = -2x^2 - 2$$

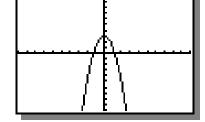
6.
$$y = -2x^2 + 3$$

What's My Curve?

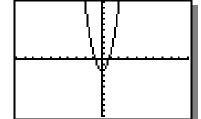
A.



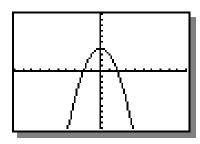
В.



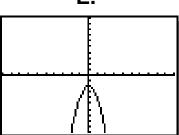
C.

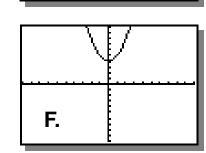


D.



E.





Part B: Describe how the graph of the graph of $y = -2x^2$ equation will compare with the graph of $y = 5x^2$ the

equation

. Then sketch the graphs.

 $y = x^2$ Part C: Write an equation for a parabola that opens down and is wider than the graph of

. Then $y = x^2$ write an equation for a parabola that opens up and is narrower than the

Wishing You Well

Show all your work!

Tonight's assignment is short, so you can have time to complete your POW write-up.



- 1. Marcus dropped a penny into a wishing well and heard it hit the bottom exactly 2 seconds later. How deep is the well?
- 2. Morgana dropped a stone from a bridge. It hit the water 1.5 seconds later. How high is the bridge? (from the water)



Galileo and Gravity

Part 1: Use your graphing calculator to record Galileo's data. Now draw the graph of the function you wrote in POW 1. You will need to rename the independent variable x and the dependent variable y to agree with the conventions of the calculator.

Does the graph of the equation pass through the points from Galileo's data?

Does the graph of the equation resemble the one you drew by hand on Day 1? Why not?

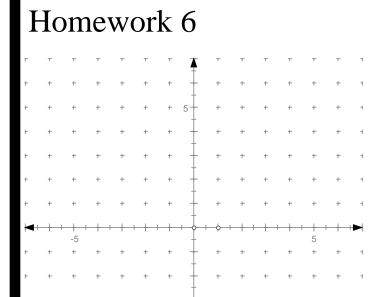
We call the graph of the equation the **abstract function**, and we call the data from Galileo the **experimental data**.

On graph paper, draw the portion of the abstract function on the interval from t = -6 to t = 6 with a colored pencil.

With a second color, trace the part of the graph that models the experimental data.

Part 2: Use your graph to answer the following questions:

- 1. When t changes from 1 to 2, what is the resultant change in s?
- 2. When t changes from 2 to 3, what is the resultant change in s?
- 3. When t changes from 3 to 4, what is the resultant change in s?
- 4. Is this evidence of a constant rate of change between time and distance, as Galileo predicted? That is, does the distance change an equal amount for every second that passes?
- 5. For positive t, as t increases, does s increase or decrease? When does s increase more rapidly: when t increases from 1 to 2, or when t increases from 4 to 5?
- 6. The change in s per unit change in t is called the average rate of change of s. Do you think that the average rate of change of s from t = 9 to t = 10 seconds will be greater or less than the average rate of change of s from t = 2 to t = 3?
- 7. Use your graphing utility to estimate the time at which the object will have fallen 100 feet.
- 8. At what time will the object have fallen 500 feet?



What's My Curve 2?

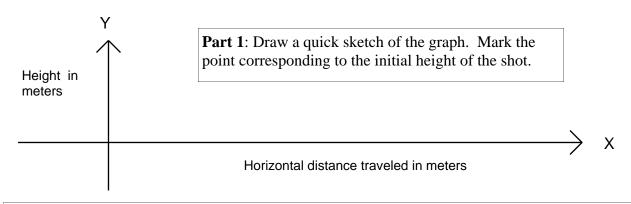
On the coordinate plane on the left, sketch a graph which fits the following description:

- the axis of symmetry is the y-axis
- a is negative
- the graph passes through the points (2,4) and (4, -2)
- the y-intercept is (0,5)

At what value(s) of *x*, does the function equal zero? (what are the x-intercepts?)

If this parabola described the path of a water baloon, what is the significance of the *x*-intercepts? (What happened to the balloon at those times?)

Shot Put



Part 2: Answer the following questions:

- 1. The initial height of the shot at the moment that it is about to leave Jane's hand is 2 meters. Verify that the equation calculates this height correctly by substituting x = 0.
- 2. Use your graphing calculator to help you answer the following questions:
 - a) How high is the shot after it has traveled 5 meters horizontally from where Jane Fleming released it? Check by substituting x = 5 in the equation.
- b) How high is the shot after it has traveled 12 meters horizontally from where Jane Fleming released it? Check by substituting x = 12 in the equation.
 - c) Find the highest point reached by the shot in its flight, and mark this point on your sketch.
 - d) Find how far from Jane Fleming the shot will land. Mark this point on your sketch.

Look Out Below!

Abby and Bing Woo are testing balloons for the carnival. Abby fills the balloons and Bing Woo tosses then out an upper-story window of their apartment. From the moment the water balloon leaves Bing Woo's hand until the moment it splats on the pavement, the height of the balloon is given by the function

$$h(t) = -4.9t^2 + 2.1t + 10.5$$
,

where h is measured in meters and t in seconds. In other words, if we know the time, we can calculate the height. Also, to a certain extent, if we know the height, we can calculate the time.

- 1. From what height was the baloon thrown?
- 2. What is the height of the baloon after 1 second? after 0. 5 seconds? (hint: substitute the time in place of *t* in the equation above)
- 3. In how many seconds does the baloon hit the pavement? (hint: what is the height of the baloon when it hits the pavement?)

Homework 7



Throwing a Curve!

Part 1: Graph the parabola which describes the path of a softball being pitched as follows:

- the height of the ball when it leaves the pitcher's hand is 2 feet
- the distance of the ball from home plate when it leaves the pitcher's hand is 28 feet
- the ball lands 35.6 feet after leaving the pitcher's hand
- the ball achieves a maximum height of 4.25 feet, 15 feet after leaving the pitcher's hand

Part 2: A strike is called if a ball crosses home plate at a height somewhere between the batter's knees and arm pits. If the ball described landed three feet behind home plate, was it a strike? Explain your reasoning.

Classwork 9

Maximum Video

Part 1: Abby and Bing Woo have decided to expand their operation into the video rental business. The following equation describes the profit they will make on a new movie, depending on *x*, the number of copies of the movie they purchase:

$$y = -(x^2) + 6x$$

Complete a table of values, similar to the one below, below by computing how much profit they will make by purchasing 0, 6 or 8 copies of a movie.



Part 2: Plot these three points on a piece of graph paper. Now draw in (with a dotted line), and label the axis of symmetry. Remember that the two roots are reflections of each other, so where would the axis fall in relation to them?

Since a parabola's vertex always falls on the axis of symmetry, you can now calculate the functions maximum or minimum without a graphing calculator. At what point does the axis of symmetry pass through the x-axis? Plug this value of x into the profit function to find out the function's maximum. (Show your work).

Locate and label the vertex on your coordinate plane. Is this a maximum or minimum point? Now draw in the entire graph of the profit function.

Part 3: Explain why this profit expression is not a linear function. What might cause Abby and Bing Woo's profit to decline after buying a certain number of copies of a particular movie.

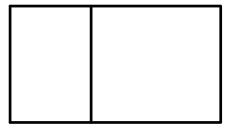
Alternate Fields



Part 1: Does a square always create the largest rectangular enclosure? Use a graphing calculator to test your hypothesis by graphing rectangular area formulae for two other amounts of fencing.

- a Draw and label the length (x) and width (in terms of x) for the two new examples.
- b For each example, write a formula, in terms of *x*, for the area (*A*) of the field.
- c What is the maximum area possible for each enclosed field?
- d What are the dimensions (the length and width) of the

Part 2: Jack still has 128 feet of fencing, but now he needs to make two rectangular enclosures one for the cow, and one for his new horse. How should he use the fencing if he wants to enclose the greatest area? Show your work.



Part 3: Jack gets wise to the fact that he can fence in more area if he uses his barn as one side of the enclosure. What's the largest rectangular area he can enclose with this new arrangement? Show your work.

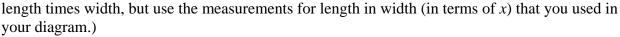
Barn

POW 2

Jack has 128 feet of fencing to make a rectangular enclosure for his cow. He could fence in an a rectangle 10 feet by 54 feet (10 + 10 + 54 + 54 = 128). What would be its area? He could also fence in an area 24 feet by 40 feet. (Check that such a field would require 128 feet of fencing.) What is its area?

The distance around the border of the fenced region is called the **perimeter**. Each of the two fields we've just considered has a perimeter of 128 feet. Draw a sketch of a rectangular field and label the width x. How could you represent the length of the field in terms of x so that the rectangle's perimeter is 128 feet? Label the other three sides of your diagram and make sure that the perimeter is 128 feet.

Write an expression for the area, A of the fenced region, measured in square feet, in terms of the length and width as expressions in x. (That is, area equals



Now, here comes the parabola part: Use a graphing calculator to graph your area formula, then use your graph to help you answer the following questions:

- What values of x make sense in the physical (experimental) problem? Explain how you came up with your answer.
- What are the corresponding values of A (area)? That is, what is the range of possible values of area that make sense in the physical problem?
- If Jack wants to enclose the largest possible rectangular area, how should he fence in his field? That is, what dimensions should he use? Describe how you used your graphing calculator to find the answer.

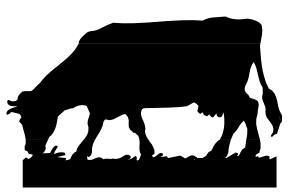
Write-Up

Be sure to answer all of the questions included in the POW description, above. Organize and present your report in our standard format:

- 1. Description of the Problem: Restate the problem.you were attempting to solve. This section should include the examples of sample areas as described in paragraph 1, above.
- **2. Process:** Based on your notes, describe what you did in attempting to solve this problem. This explanation should include the rectangular diagram and how you came up with the abstract area formula.
- **3. Results**: State all your findings in an organized manner. This should include:
 - a. the answer to the problem, and how you know your solutionit is correct
 - b. graphs of both the abstract function and the area function for Jack's field
 - c. answers to all other questions, above
 - d. any other observations or conjectures you have made

Jack's Field





Balloon Toss Revisited

Abby and Bing Woo decide to hold a carnival to express appreciation to their customers. One of the carnival booths will be a water balloon toss, where contestants pay for the opportunity to toss water balloons at local dignitaries.

Mr. Levy, a local math teacher, agrees to be one of the victims - on one condition: He doesn't mind getting wet, but is afraid that some of his Algebra students would take the opportunity to inflict bodily harm. In order to keep the balloon toss safe, Bing Woo suggests that the contestants should be made to toss the water balloons through a raised hoop. This would keep any of the contestants from throwing the water balloon overhand at Mr. Levy.

Your task is to design the booth for the water balloon toss. Your design should include both a drawing of what the booth will look like from the front, and an overhead diagram showing all of the following measurements:

- the dimensions of the largest (greatest area) booth possible from 30 feet of fencing
- the distance between the contestants and Mr. Levy's chair
- the height of the hoop
- the distance of the hoop from the contestants and Mr. Levy

Your Assignment

Pretend that your group is a business consulting team. The Woos have come to you for help. Not only should you present your design to them, but you should explain clearly how you know that your design will satisfy their concerns and insure that the greatest number of people will hit Mr. Levy.

Your presentation should include the following:

- a written explanation of your solution, with smaller versions of your designs
- two large-size diagrams with clearly labeled measurements
- any other graphs, charts, equations or diagrams that are needed for your explanation

Beginning Portfolio Selection

The main problem for this unit, *Balloon Toss*, explored how **quadratic functions** and their parabolic graphs relate to a real world situation.

- 1. What is a quadratic function? What features of a quadratic function determine the following aspects of its graph: parabolic shape, orientation (opening up/down), maximum/minimum, y-intercept, x-intercepts, roots/zeros. How do these features relate to real world situations?
- 2. Choose three activities from the unit that helped you understand the relationship between a quadratic function and its graph. Explain how each activity you selected, helped you understand this relationship.

Note: This homework assignment will be included in your portfolio.

Balloon Toss Portfolio

Now that *Balloon Toss* is completed, it is time to put together your portfolio for the unit. This has three parts:

- Writing a cover letter summarizing the unit.
- Choosing papers to include from your work in this unit.
- Discussing your personal growth during the unit.

Cover Letter for Balloon Toss

Look back over *Balloon Toss* and describe the central problem of the unit and the main mathematical ideas. Your description should give an overview of how the key ideas were developed in this unit and how you used them to solve the central problem.

Selecting Papers from Balloon Toss

Your portfolio from Balloon Toss should contain the following items:

- Homework 14: Beginning Portfolio Selection Include the three activities from the unit that you selected in Homework 14: Beginning Portfolio Selection along with your written work about those activities.
- A *Problem of the Week* Select one of the two POW's you completed during this unit (*Galileo* or *Jack's Field*)
- Other quality work Select two other pieces of work that demonstrate your best efforts. (These
 can be any work from the unit POW, homework, classwork, and so on.)
- Balloon Toss Revisited Include your report from this problem.

Personal Growth

Your cover letter for *Balloon Toss* describes how the unit develops. As part of your portfolio, write about your own development during this unit. You may want to address the following:

- How you feel you progressed in areas of: working together with others, using a graphing calculator, writing about and describing your thought processes
- What you feel that you need to work on and how you might work on it.

You should include here any other thoughts about your experience with this unit.

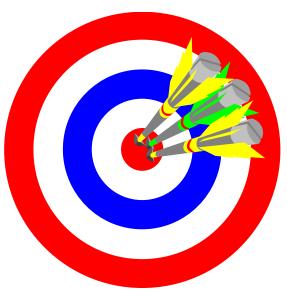
In - Class Assessments

Problem 1: An arrow shot from a bow follows a path that is very close to parabolic. An archer, therefore, aims the arrow above, rather than straight at, the target. The flight of the arrow illustrated below might be represented by the function

$$h = -0.00033x^2 + 0.036x + 4.75$$

where h is the height above ground level and x is the horizontal distance from the archer's hand. (Both h and x are measured in feet.)

- a How high is the arrow at the moment it leaves the bow?
- b How high is the arrow 25 feet after leaving the archer's hand? after 75 feet?
- c When does the arrow achieve a height of 5 feet?
- d What is the greatest height the arrow reaches during its flight?
- e If the center of the target is 4 feet above the ground and 120 feet from the archer, and if the bull's eye is 9.6 inches in diameter, does the archer score a bull's eye? Does the arrow land above or below the center of the target? above, below, or within the bull's eye area? Explain and give mathematical evidence for your answer. (We will assume that the arrow does not veer to the right or left as it flies. This is not entirely reasonable, of course, but the sideways motion would make a seperate problem.)
- f Re-write the arrow function in factored form



Problem 2: Jack has 15.4 meters of fencing with which to build a rectangular-shaped holding pen for his sheep. The pen is to be located next to a corner and wall of an existing shed as shown in the diagram below. Fencing is not needed along the shed walls.

Use the information above and in the diagram to answer the following questions:

- a Prove that the dimensions of the pen, in meters, (the length and width) are x + 2.8 and 12.6 2x
- b Write a formula, in terms of x, for the area (A) of the pen.
- c Draw a graph which represents the abstract area function you wrote in question b. Label the coordinates of the vertex of the graph.
- d What is the maximum area possible for the pen?
- e What are the dimensions of the length and width of the pen that gives the maximum area?
- f What is the domain of the area function; that is, what values of x make sense in the real world situation?



